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After engaging in various commercial activities, Mr. Davis joined the government service in 1940 as a project engineer, advancing later to Division Chief, and Laboratory Chief. His work in these capacities included electronic research and development, particularly in the fields of navigation, guidance, and radar. In 1952 he was appointed Scientific Director of the Rome Air Development Center at Griffiss Air Force Base, Rome, N. Y. He is responsible for the technical direction of a large-scale program in the development of ground electronic equipment and systems.

During 1955 he was an instructor in graduate courses in electrical engineering at Columbia University, New York. He is now a member of a study group which assists the staff of the Secretary of Defense in special studies involving the evaluation of weapon systems.

He is a member of Sigma Xi, the American Physical Society, the American Association for the Advancement of Science, the American Ordnance Association, and the Institute of Navigation.

Guest Editorial

AFTER publication of two excellent issues of the PGMIL TRANSACTIONS, the editors decided to prepare an edition which would stress Space Technology, covering those aspects which might be of general interest to all members of the IRE and, more specifically, to those who because of their interest in Military Electronics joined the PGMIL.

Not that it is necessary to stress the Space Age. A dramatic illustration of this was noted recently in a local essay contest held for teenagers. The topic of the essay was "The Space Age and What It Means to Me." I considered this to be not quite the most original topic and, acting as a judge, was prepared for the usual type of essay that such an imaginative subject would produce. Quite surprisingly, therefore, I and the other judges found, buried among run-of-the-mill papers containing fanciful ideas and trite generalities, several essays expounding ideas which may, with considerable hazard, be termed exceedingly delightful fantasies. Other contestants concentrated upon the realities of life on this earth and stressed the fact that old Mother Earth will be our home and, by far, our most important home for a very long time.

The lesson we can learn even from these children is that although we may venture mentally into space at a pace faster than is permitted by our present hardware, the bulk of our work will remain in a region surrounded by and permeated with our earth fluids of various physical characteristics. True, military electronics will lead the way into an expanding space technology, but even in military electronics there is much to be done "on earth as it is in heaven."

Why, then, a Space issue? Primarily, it is felt that the military engineer must be and is the most versatile among scientists. Secondly, in his daily activities, he may not be exposed to newer concepts in other technologies which may affect his future work. A knowledge of the dynamics of satellites and planets and of the solar system and the universe is and will continue to be important to the military engineer. Finally, in recognition of the innate curiosity of the electronic engineer and, to the satisfaction of that curiosity, that thirst for knowledge beyond his own specialty, this issue is dedicated.—Harry Davis.

Our Interest in Space and Its Technology*

H. GUYFORD STEVER†

Summary—The urge to learn more about the space surrounding the earth has been strong throughout mankind's recorded history, the incentive coming sometimes from primitive religion, sometimes from scientific curiosity, and ofttimes from a love of natural beauty. Today we can add to these the incentive of our newly acquired capability of penetrating into this space with instrumented and manned vehicles. We already know much about the solar system and a little about the universe beyond. We have some knowledge of the physical existence and the motion of the planets and their satellites, the asteroids, the comets and the meteors; the composition of these bodies is also somewhat known. The nature of the electromagnetic and particle radiations from the sun and extra solar system sources are less completely known and will be the continuing objective of some of our early scientific explorations of solar space.

In this first year of successful space flight, a large number of people, scientists and nonscientists alike, have studied enthusiastically the space surrounding the earth and, in fact, the entire cosmos. Much of what they have learned is knowledge inherited from the past, and not knowledge acquired during this first year of space technology nor even in recent decades. They have realized that this generation is not distinguished by a greater interest or by a disproportionately large contribution to knowledge about the space around our earth. We are, however, the first to possess the capability to explore it by inhabited or uninhabited vehicles under our control.

The beauty and inspiration of the heavens have been shared by all men in the past. Possibly the heavens were more important to the past than to the present. Certainly early man's attention to the heavens in relation to his primitive religion gave him a motivating interest which we do not have today. Early man assigned his primitive gods to homes on the planets, the wanderers, in the heavenly skies. The urge to observe, to record, and to predict the motion of the planets was a strong one because it pertained to religion in early civilizations, particularly those in the eastern Mediterranean, Babylonia and Egypt, and this motivation persisted through the middle ages until the postulation of the heliocentric theory of the universe and the birth of modern science.

The beauty and the mystery of the cosmos fired man's creative imagination; ancient literature and folklore is rich in the ideas and stories of space flight and space travel. In ancient days, when the space between heavenly bodies was thought to be filled with air, space flight was just an extension of bird flight. One of the early examples of this type of tale is the legend of Daedalus and his son Icarus, who flew on wings constructed of wax and bird feathers in their attempt to escape from imprisonment on Crete to return to their home on Sicily. Icarus flew too close to the sun, melted the wax holding his wings, and fell to his death into the sea off Greece. Another story was of King Bladud, tenth legendary king of England, founder of Bath, and father of the King Lear of Shakespeare's tale, who was said to have been killed in a flying attempt. Other authors, particularly Lucian, a second-century Greek satirist, wrote of very imaginative trips among the planets and stars. Cicero, the great Roman lawyer and statesman, in his book "Die Somnium Scipionis," described in a dream of Scipio a trip which started in the ancient city of Carthage and ended with a completely cosmic view of the universe. It is interesting to note that the great nonscientific minds of those days were well informed of the latest concepts of the universe, which were held by the natural philosophers; it is particularly interesting that Cicero mentions a "milky circle" which has in it "stars which we will never see from the earth, all larger than we have ever imagined." Also, Plutarch, best known as a great biographer, wrote a description of the physiognomy of the moon in his "De Facie in Orbe Lunare."

Tracing the many threads of man's interests in the cosmos from the early child-like curiosity of simple people to the more complicated observations, calculations, and predictions of stylized religions, on to the disciplined theorizing of the early Greco-Roman natural philosophers, to the unification of the basic principles during the Renaissance and the birth of modern science, to the deepening understanding of astrophysicists of the present day, we see that one of the most compelling and constant reasons for learning more about the space surrounding the earth is so that it could be used as an extraordinary scientific laboratory. The fund of observational data, which started with the ancient religions, was carried on by the astrologers and by the early astronomers, including Copernicus, Kepler, and Brahe, was inherited by those great physicists, Galileo, Newton, and others, and served for them as superb scientific data. The roots of the laws of motion and the law of gravitation are in those studies. The early development of scientific instruments, such as the telescope and the spectroscope, owe much to their use in the studies of the motion and the nature of the heavenly bodies.

Our current interest in space technology, or at least that part of space technology which has to do with space flight, is of course the solar system. To be true, the solar system is a minute part of the universe, but we will concentrate on it, for it can now be reached by vehicles designed on the basis of our current technology.

Now that mankind has this capability, there is a

* Manuscript received by the PGMIL, September 8, 1958. Many of the ideas and descriptions in this article are to be published in less abbreviated form as a chapter in the forthcoming book, "Space Technology," Howard Seifert, ed., John Wiley and Sons, Inc., New York, N. Y.

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whole new set of reasons why man will push into outer space. Many of these have nothing to do with science. One motivating force is curiosity, which is strong in scientists, but another more universal motivation is a love for adventure Some men have a love for adventure and a natural desire, which they cannot explain but only must satisfy, to explore new things. Often what they explore eventually proves to be of great importance to mankind, but that is not the reason they explore it. They are challenged simply because it is something to do which satisfies them. When this exploration of the solar system has taken place, and when men are able to travel to outer space, who knows what mankind will do with this capability? If one takes a very long-term view, there is a certainty of the eventual use of other parts of the solar system for gainful purposes; our generation cannot foresee these uses in detail.

What is the state of our knowledge of this cosmos which we have inherited from these thousands of years of studies by men? Most of us who were trained in modern science and engineering have a rather stand-pat concept of the solar system and the universe beyond the solar system. It is well to reexamine this, for already in this first year of space exploration some new data have been changing and adding to our ideas about the space around us.

The Solar System

Normally we think of the solar system in terms of the sun and the nine planets, together with their various moons. These planets in their almost circular, elliptical trajectories around the sun, together with the sun itself, account for most of the material of the solar system. The sun itself, with its diameter of 865 thousand miles, contains more than 99 per cent of all the matter of the solar system. Most of the rest of the matter is contained in the planets, particularly Jupiter and Saturn. Uranus and Neptune are respectable, medium-sized planets, while Earth, Venus, Mars, Mercury and Pluto are assigned to the smaller group. In numbers, however, there are many more bodies, including the asteroids, the comets, the meteors, and interplanetary dust. In addition, there is a great deal of electromagnetic and particle radiation which is an important part of the content of the solar system.

The Sun

The sun, with its more than 99 per cent of the mass of the solar system, naturally falls at the center. It is the source of a great deal of energy radiated primarily in the form of electromagnetic energy but, in part, in the form of particle radiation. This radiation greatly affects all that happens in the solar system, with the exception of the mechanical motion of the planets and other bodies about the sun, which is, of course, determined primarily by the universal law of gravitation and Newton's laws of motion, and depends upon the inert mass and the velocities of the bodies.

The sun has a diameter more than 100 times that of the earth, is 93 million miles away, and has a third of a million times more mass than the earth. The center of the sun had a density seven times that of lead, and in the center there is a pressure possibly of a billion tons per square inch and a temperature of 20 million degrees Centigrade. In this atomic furnace the nuclei collide with such velocities that transformation from atomic species to others is made. There are complex nuclear reactions which in their total use up the most plentiful element in the sun, hydrogen. From hydrogen, heavier nuclei are built and there is, as a result of this fusion, a release of enormous energies. A calculation has been made that 564 million tons of the sun's hydrogen are transmuted every second into 560 million tons of helium and 4 million tons of energy. This energy flows out to the surface and is radiated primarily in the form of electromagnetic radiation. Here on earth that radiation keeps life going, yet we intercept of the order of a billionth part of all the sun's radiated energy.

The surface of the sun, the so-called photosphere, is a gaseous envelope perhaps 200 miles thick. In the outer portions of this atmosphere the gases shine only faintly; at the bottom of the atmosphere they are intensely bright. The density of this thick atmosphere is not great, possibly a millionth of the density of air at sea level here on the earth. The pressure, in fact, is only about one-fifth of our sea level pressure, but of course the temperature is very high, 6300 degrees Centigrade. At the top of the photosphere the temperature drops probably to 1400 degrees and the density and pressure are decreased correspondingly. Furthermore, the gas at the top is quite transparent to the radiations from the lower levels of the photosphere.

Outside the photosphere is the chromosphere which is a turbulent, rarified atmosphere extending 5000 miles or more above the photosphere. It is in this chromosphere that the great solar storms and prominences, which appear like great protruding flames, occur. Outside this chromosphere is a very faint corona reaching millions of miles off in some directions. The speed of molecules in this corona is very great, indicating temperatures possibly 500 thousand degrees absolute. As viewed from a great distance, this corona might even be considered to stretch out to beyond the closest few planets.

The electromagnetic and particle radiation from the sun is not entirely steady; irregularities of this radiation cause perturbations in many phenomena throughout the solar system.

A good example of this can be seen from the effects which occur when a large solar flare occurs on the surface of the sun. These solar flares seem to be associated with sun spots, which are the black spots on the face of the sun, the great vortical motions within the sun's atmosphere; the sun spots cross the sun from east to west and vary from year to year in intensity and number, apparently with an eleven-year half-cycle.

Solar flares dissipate immense energies in a few minutes. Almost immediately after a solar flare occurs, the shortwave radio communication blacks out on the earth, at least on the daylit hemisphere of the earth. Also, solar radio noise increases. In a few minutes the solar flare will have generated light which is many times as bright as the surrounding regions of the sun; then the flare will die down in a few hours.

There are also some delayed effects of solar flares. About a day after the occurrence of a solar flare, a magnetic storm will occur which will disrupt the magnetic field of the earth. It will trouble wireless communications for days thereafter. Also, about a day after a solar flare the auroral activity will increase greatly.

The current theory concerning the nature of the sudden release of energy in solar flares is that in the tenuous gas in the sun's chromosphere the moving electrically-charged particles of gas distort the magnetic field lines and store energy in the twisted lines, the pressures of which are resisted by the pressures in the gas, until finally a great amount of energy is stored. Suddenly an instability sets in and all this energy stored in the electromagnetic field is dissipated in a short time and radiated primarily in the form of electromagnetic energy but partially in the form of ionized particles and electrons.

Terrestrial Space

Clearly from the above description the particle and electromagnetic radiations from the sun have a great influence on the space immediately surrounding the earth. Many of the particles and radiations are absorbed in the upper reaches of the earth's atmosphere. This atmosphere acts as a protective blanket for the plant and animal life on the earth. The harmful X rays and ultraviolet rays from the sun are absorbed, as are most of the cosmic rays and meteors. Many of the effects of the solar flare result from a change in the electrical layers in the upper atmosphere which in turn change radio transmissions. In addition, the circulating electric currents outside the denser portion of the atmosphere which are responsible for part of the earth's magnetic field are changed. The incoming particles from the sun, as they spiral in around the earth's magnetic field lines, also cause ionization in the upper atmosphere leading to the radiations which are associated with the earth's aurora.

Asteroids

There are thousands of asteroids, or little planets, which have already been detected. The first and largest one, Ceres, was discovered by Piazzi on the first and second days of the nineteenth century. A few years later Pallas, Juno, and Vesta had also been found and plotted. By the middle of the twentieth century more than fifteen hundred of these asteroids were tabulated and registered.

The typical orbit of an asteroid is almost circular and is located part way between Mars and Jupiter. The numerous asteroids which occur in this region seem to indicate they were formed by the violent destruction of a planet or, possibly, they are evidence of a planet which never quite coagulated. Modern theory seems to indicate that the latter is more likely. There are some asteroids, however, with orbits quite different from the typical asteroidal region. Hidalgo, for example, has a highly eccentric orbit in which its perihelion is closer to Mars than the typical asteroid orbit, and its aphelion stretches almost out to the orbit of Saturn.

There have been many estimates of the number of asteroids which have existed, and attempts have been made to determine the range of sizes. There seem to be about two hundred with an absolute magnitude of seven or less and the estimates range up to 30 thousand to 80 thousand with magnitudes greater than 19. Possibly there is as much matter in the form of asteroids as there is matter in the earth, though more likely the mass of the moon is a good comparison. The large asteroid Ceres has a diameter of almost 500 miles which, as you see, makes it not very different from the size of the moon, Ceres being one-fifth of the diameter of earth's moon and, in fact, one-seventh the diameter of the planet Mercury. Only one of these asteroids, Vesta, is visible with the naked eye; this is not because it is larger than Ceres, but because it has a greater surface reflectivity.

Some of the more recently discovered asteroids have orbits passing reasonably close to the earth. One such asteroid, Eros, actually passed inside the oribit of Mars and, in its closest approach, came within 15 million miles of the earth. Apollo, Adonis, Hermes, Icarus, and others have actually come closer, with Hermes passing within 500 thousand miles of the earth, or just about twice the distance from the earth to the moon. Asteroids which come close to the earth, however, are rather small in size.

Comets

Comets in recorded history have occasionally been spectacular. Our generation has not yet been treated to the most spectacular comets as have previous generations, but even the less impressive ones are interesting. Comets have been recorded for centuries. For example, Halley's comet was recorded in Japan and China every time except once since 240 B.C., appearing at approximately seventy-five year intervals. Apparently Halley's comet is the one pictured in the ancient Bayeux tapestry; it was also considered to be an ill omen for the Saxons in the battle of Hastings in 1066 A.D.

The orbits of comets vary greatly, both in their eccentricity and the angle between their orbital plane and the ecliptic, and even in the direction of motion about the sun. Some comets have almost circular orbits while others have orbits of great ellipticity going from perihelion close to the sun to aphelion far, far out beyond the farthest planet. Some have orbits which are almost hyperbolic; in fact, some orbits can be perturbed by a

proper combination of gravitational influences by the planets, particularly Jupiter and Saturn, to change their orbit slightly from elliptical to hyperbolic, and so become lost to the solar system.

In appearance comets vary a great deal, but a typical comet appears first in the sky as a diffused spot of light. As it comes closer to the sun it seems to grow a tail pointing away from the sun. Some of these tails are extremely long. For example, the tail of Halley's comet in 1910 was about 30 million miles long when it passed perihelion and reached a maximum length of about 100 million miles somewhat later. This distance is greater than the distance from the sun to the earth. There is usually a bright nucleus in the center of the coma which is a diffused area about the nucleus. Tails are varied in shape and size; sometimes they have multiple structures.

The composition and structure of comets, in a theory which is of relatively recent origin and due largely to Whipple, is a mass of material loosely amalgamated, the material consisting of small solid particles frozen together in an ice of water, methane, ammonia, and possibly cyanogen and carbon dioxide. The nucleus of comets is not necessarily very large, Halley's having been estimated to be as low as 1500 feet in diameter and certainly not more than a mile. When these comets are far away from the sun they consist of the loosely amalgamated particles in the frozen ice. Possibly in the farthest reaches of the solar system they pick up additional matter by collision with particles and gaseous clouds. As they approach quite close to the sun, the sun's heat melts the ice and soon it evaporates; vapors are emitted in a very irregular process. Presumably the emitted vapors and particles make up the coma and tails of these comets. The reason why the tails are always away from the sun is not completely understood, but is due probably to a combination of light pressure, the collision of the tail particles with particle emanations from the sun, as well as electromagnetic forces.

Meteors

There are many solid bodies in the solar system which are much smaller than the comets and asteroids. In fact, many of the small particles which make up comets are in the class of these smaller bodies. When these particles hit the atmosphere they form the typical meteor trail. Possibly a billion meteors strike the earth's atmosphere each day; most of them are small specks of material which burn up in the air rapidly and are not visible. A very small number are large enough to penetrate the atmosphere and hit the earth somewhere. Once in a great while a very large one hits the earth, causing a great deal of damage, creating a great crater, and destroying things far away from the crater by the blast caused by high-speed passage through the air. The speed of these meteors varies a great deal, the average being 30 miles per second. Most of them vaporize completely while up in the atmosphere near the ionosphere, some 40 to 60 miles above the earth. The incandescent gas in the trail of the meteor is often spectacularly visible.

Many meteors are independent of meteor showers, but a large number of them are associated with showers which in turn are associated with comet orbits. Apparently the meteors associated with comet orbits are the loose solid particles which have been separated from the comet. Some comets which have completely disappeared still have associated with their orbits a number of particles which, when the earth's orbit crosses that of the comet, creates a meteor shower. A very spectacular meteor shower occurred in 1833, the Leonid. It was also spectacular in 1866, but in 1899, the next time that the orbit of the earth crossed the orbit of the comet in a position where many particles normally were, the showers seemed to have been deflected by the gravitational pull of other planets. There are other well known meteor showers, the Perseids, the Lyrids, the Andromedes, and others.

Interplanetary Dust

There is a great deal of material in the solar system which is best described as interplanetary dust. This can be observed by spectroscopic and optical observation. It creates a very faint glow in the night sky and is particularly concentrated in the plane of the ecliptic. It is thought that much of this dust, possibly supplied from the outer regions of the solar system and carried in by comets, is falling in toward the sun.

Life in the Solar System

Throughout history there has been great hope of finding life within the solar system on planets other than earth. In ancient days the gods were thought to live on the planets. Great men dreamed and wrote of civilizations on the planets and stars. Even as late as 1877 when Schiaparelli discovered the canals, the straight-line markings, on the face of Mars, there was a flurry of excitement which lasted for decades over the possibility of finding intelligent life on Mars.

Today we are much less expectant. The conditions which we know from observations to exist on the various planets leads us to expect that intelligent life does not exist on other planets. Possibly on Mars there is a very simple plant life. Other than that, conditions are not very good. With respect to space travel, this is one of our greatest disappointments, because the possibility of meeting other forms of animal life would be a tremendous spur to the imagination, and a driving force to achieve manned space flight quickly.

The Universe

The universe stretches far beyond the solar system. Our sun is out toward the rim of an average spiral nebula, the Milky Way, which is made up of possibly 100 billion stars and great dark clouds of interstellar dust and gas. We are about 26 thousand light years (a light year is about 6×10^{12} miles) from the center, and

we will rotate about it in 200 million years. The whole diameter of the Milky Way is from 100 thousand to 300 thousand light years and it is 25 thousand to 40 thousand light years thick.

Beyond our galaxy there are countless galaxies and clusters of galaxies stretching out at least to the limit of vision with the largest telescope, about two billion light years.

When one considers that we are now straining to get our space vehicles out a few hundred miles from the earth, it becomes clear that mankind will not soon run out of incentive. In fact, to travel to the nearest star is a job far beyond our current capability. That star, Proxima Centauri, is about four and one third light years away. The current obstacle to man's travel to such a distance seems to be a limited life span and the limiting speed of travel, the velocity of light. There are some practical rocket motor problems, as well as the problem of storing sufficient energy per unit mass of fuel to attain anywhere near the velocity of light. Perhaps when we achieve nuclear reactions which result in 100 per cent conversion of fuel mass into energy, instead of the one per cent efficiency of the fusion process and the 0.1 per cent efficiency of the fission process, our prospects for travel to the nearest stars will be sufficiently improved for a try.

In the meantime here within the solar system there is first the challenge to learn more about it by scientific and manned exploration. Not far behind such a challenge comes that of putting our space capabilities to use. Along this line the only nonmilitary proposal which so far has evinced considerable interest is that of establishing communication relay stations on satellites. Fortunately this practical application is attainable with relative ease.

Beyond the scientific, military and simple nonmilitary applications of our space travel capability, most of us are confident that as we further the development of the capabilities we inherited from past generations of science and technology, there will unfold many practical uses which we can will to the future.

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Some Aspects of Astronautics*

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EDITED BY P. SWERLING†

Summary—**This paper is mainly concerned with four general topics of importance in astronautics:**

1) Basic laws of celestial mechanics. The subjects covered are: Kepler's laws and their Newtonian redevelopment, the orbital elements, and perturbations.

2) Lunar and interplanetary flights. A typical earth-moon transit trajectory, computed by automatic machine, is discussed. Guidance accuracies required for lunar impact are illustrated. Circumlunar flights, lunar satellites, and interplanetary flights are also briefly discussed.

3) The space environment. Among the subjects covered are: the distribution and characteristics of dust and meteoric material in the solar system; asteroids; comets; molecular, atomic, and subatomic particles in space; the possible lunar atmosphere and ionosphere; extraterrestrial radio noise; and the magnetic fields of the earth and sun.

4) Scientific experimentation in space. Useful subjects for experimentation are: refinement of our knowledge of basic constants such as the value of the astronomical unit; observation of the atmospheric and surface conditions of the moon and of our neighbor planets; increased observation of matter and radiation in space.

INTRODUCTION

THOSE who first venture into space will, unlike earlier navigators pushing across unexplored seas, find that much of the region to be traversed has already been charted and something of the character of both space itself and potential destinations in space is known. But there is always the difference between indirect knowledge and first-hand experience, and this difference undoubtedly will show up trenchantly on the first flights into space.

Before entering into details, a few important basic differences between space environment and terrestrial environment should be mentioned and kept in mind in our discussions of space. First, the configurations of bodies in space are never static; relative distances are always changing. Second, the description of the solar system in terms of distances alone is inadequate. The astronaut must think also in terms of all the orbital elements: the eccentricities, the inclinations, the nodes, the epochs, and the perihelions as well as the semimajor axes. The third general difference is the relation between energy expended and distance traversed. In space this will be completely unlike anything in terrestrial experience. Fourth is the matter of the scale of space. It is always most difficult to visualize the tremendous distances involved. A fifth difference is that space travel will be performed in vehicles which are intermediate in size between the small particles in free space and the massive planets. While the motions of the latter are influenced only by gravitational forces (Newtonian and relativistic), the small particles are, in addition, subject to magnetic, electrical, and radiation forces. It is to be expected that future space ships will, as intermediate-sized bodies, experience to some extent the effects of all of these forces.

There are many possible ways to classify space-flight activities, such as powered and ballistic, manned and unmanned, scientific and military, etc. One of the most useful of these ways is to order space flights by flight mission.

The main categories of activities of general interest are earth satellites, lunar flights, and interplanetary flights. Let us first consider the gross dimensions of these flight classes.

In the case of satellites the distance parameter of interest is orbit altitude. This can range from about 100 miles to about 1,000,000 miles. Beyond about 1,000,000 miles from the earth, the sun's field will disturb the vehicle to such an extent that the term "earth satellite" tends to lose its meaning. The time parameter of interest is orbit period; this will range from about $1\frac{1}{2}$ hours to about 8 months.

Lunar flight distances are, of course, roughly the distances from earth to moon—about 240,000 miles. Flight times will range generally from about one day to one month or more.

The interplanetary theater starts at a distance from the earth of about 1,000,000 miles and extends to the orbit of Pluto, nearly 5,000,000,000 miles at maximum displacement. Flight times would fall roughly in the range of one month to 50 years.

In the category of satellites we have two principal types: nonrecoverable satellites, and recoverable satellites.

The nonrecoverable earth satellite is now a familiar system. Its feasibility has been established beyond any reasonable doubt.

The recoverable satellite is so contrived that all or part of the satellite is perturbed by an on-board rocket so that it returns to the surface of the earth.

The lunar flight category can be broken down into the following principal missions:

1) Impacts on the moon.
2) Nondestructive landings on the moon.
3) Artificial satellites of the moon.
4) Circumlunar flights.

Interplanetary flight would, in turn, involve execution of the following:

1) Impact on the planetary surface. (Impact here has its usual meaning—a destructive collision.)
2) Land intact on the planetary surface.

* Manuscript received by the PGMIL, July 23, 1958.
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3) Set up an artificial satellite of the planet
4) Orbit around the planet and return to earth.
5) Set up interplanetary space buoys.

Basic Laws of Celestial Mechanics

Celestial mechanics, which is the basis for the determination of orbits or trajectories in space, is usually thought of as beginning with the publication of the "De Revolutionibus," by Copernicus, in 1543, although the subject has important roots nearly two thousand years before this date.

A second major step was made by Kepler in his discovery of the laws of planetary motion (see Fig. 1):

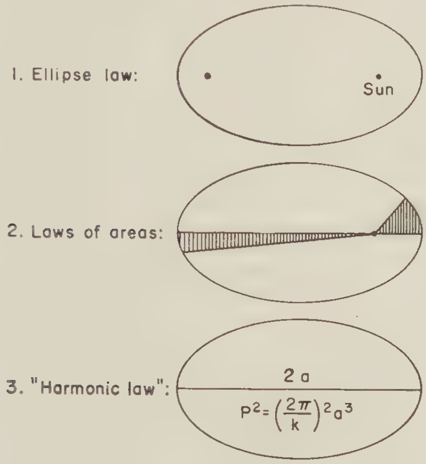


Fig. 1—Kepler's laws.

1) The orbits of the planets are ellipses with the sun at one focus.
2) The line joining the planet to the sun sweeps over equal areas in equal intervals of time.
3) The square of the period (P) of a planet is proportional to the cube of its mean distance (a).

In Kepler's third law the period, usually designated by P, is the length of time it takes the planet, comet, or today, satellite, to travel around its orbit. The mean distance, a, is sometimes called the semimajor axis (see Fig. 1) and is in fact the average of the greatest and least distances, the perihelion and the aphelion distances in heliocentric orbits, or the perigee and apogee distances in geocentric orbits.

With his law of universal gravitation and his laws of motion, Newton was able to rederive the Keplerian laws of planetary motion. In doing so he found it necessary to modify them in significant ways.

1) Kepler's laws define the motion of a planet exactly only if it is alone with its sun in the universe. Every other object in the universe will disturb the simple Keplerian motion, producing what we call perturbations. In Fig. 2 we see the effect of an extremely large perturbation. A comet or minor planet is traveling around the sun in Keplerian orbit A. One time, when it is crossing the orbit of Jupiter, it finds Jupiter nearby, at J. Jupiter's attraction is momentarily very large, causing the disturbed object to be hurled off toward the sun in a new direction. After it is safely past Jupiter the sun's attraction again becomes predominant, and the object thereafter travels in orbit B. Of course the attraction of Jupiter is never negligible, and so is progressively changing the orbit, though more gradually than in the illustration.

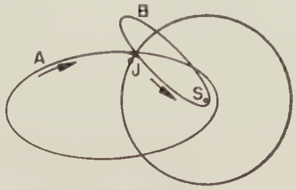


Fig. 2—Perturbations.

2) Newton showed also that Kepler's laws would be exact for a two-body system only if the two bodies were homogeneous in spherical concentric layers. Because of its rotation, the earth is not perfectly spherical, but bulges out at the equator. The bulge will introduce perturbative forces on the moon or an artificial satellite. These cannot be resolved into a single force acting from the center of the earth. The bulge perturbations of the orbit of an artificial satellite are much larger than those caused by the sun or the moon.

Other forces that may be treated as perturbations, when they are not too large, include thrust, drag, and other aerodynamic forces, and possibly electromagnetic forces, radiation pressure, and the modifications introduced into the gravitational field by Einstein mechanics.

3) Kepler's laws, in Newton's redevelopment, emerge as integrals of the two-body problem. There are many other useful integrals, of course, and at least one of them has such a simple form as to be especially useful in the solution or interpretation of orbit problems. It is the *vis-viva*, or energy integral, which expresses the fact that the sum of the kinetic and potential energies is constant:

$$V^2 = k(m_1 + m_2)\left(\frac{2}{r} - \frac{1}{a}\right).$$

In this equation, k is the gravitational constant, m_1 and m_2 are the masses of the two bodies, a is the semimajor axis, V is the velocity, and r is the distance from the focus.

4) Newton found that in the two-body problem the ellipse was not the only possible orbit. Parabolas and hyperbolas were also possible orbits. The addition of parabolas made it possible for Newton to show that the comets, which travel in nearly parabolic orbits, obey the same laws as the planets.

For illustration, let us suppose that a circle is the orbit of a satellite somewhat above the surface of the earth, with a velocity just under 5 miles per second. If the velocity were cut to 3 miles per second, the satellite would fall inward on a smaller ellipse until it encountered the surface of the earth. Conversely, if we increased the velocity of our projectile to 6 miles per

second, we would find that it would rise up on a larger ellipse. If next we think of the velocity as increased to 7 miles per second along the same horizontal tangent, we find that the object will travel off on a parabola, never slowing down enough to return. This critical velocity, approximately 7 miles per second, is called the "velocity of escape." It is the same whether the direction of projection is horizontal, vertical, or some angle in between. A velocity of 8 miles per second would carry the projectile off on a hyperbola; still higher velocities, on more nearly rectilinear hyperbolas.

The so-called velocity of escape applies strictly only if we neglect all other forces in the field. With a velocity of 7 miles per second a projectile would escape, at least temporarily, from the earth, but not from the sun. As it receded from the earth, in any direction, its velocity would quickly drop off nearly to zero. But with its geocentric velocity nearly zero, its heliocentric velocity would be nearly the same as that of the earth, *i.e.*, $18\frac{1}{2}$ miles per second in a direction approximately at right angles to the direction of the sun. And so the escaped vehicle would take up a nearly circular orbit around the sun closely approximating that of the earth.

The Orbital Elements

A two-body orbit, as illustrated in Fig. 3, is specified by six constants, called the "elements" of the orbit. Three of these elements have to do with the orientation of the orbit in space, and require that we specify arbitrarily a reference plane, and in that plane a reference direction. For geocentric orbits we use the plane of the earth's equator and the direction of the vernal equinox. (For heliocentric orbits the reference plane is usually the ecliptic plane, *i.e.*, the plane of the earth's orbit.) The intersection of the orbit plane and the equator plane, in the geocentric case, is called the line of nodes. The ascending node is the point at which the object passes from the south side to the north side of the equator, and the descending node is the point at which it passes from north to south. Three angles that may be used for orientation elements are, then:

Ω = the longitude of the node, or the angle between the directions of the vernal equinox and the ascending node,

i = the inclination, or the angle between the two planes,

ω = the argument of perigee, or the angle in the orbit plane between the direction of the ascending node and the direction of perigee.

The remaining elements specify the size and shape of the orbit and the time at which the orbit is at some specified point. These may be:

a = the mean distance or semimajor axis.

e = the eccentricity, which may be defined as the distance from the center of the ellipse divided by the semimajor axis.

T = the time of perigee passage.

These six constants often are replaced by others in part or altogether. For example, orientation unit vectors, $\boldsymbol{P}$, directed to perigee, $\boldsymbol{Q}$, parallel to velocity vector at perigee, and $\boldsymbol{W}$, perpendicular to the orbit plane and making up a right-handed system with $\boldsymbol{P}$ and $\boldsymbol{Q}$, often are used as orientation elements in place of Ω, i, ω.

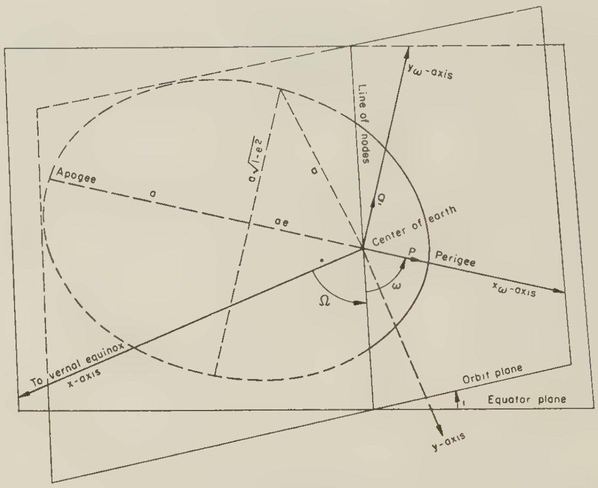


Fig. 3—The elements of an orbit.

Perturbations and Precision

Two-body orbits and elements are very useful if the perturbing forces are not prohibitively large. Often the perturbing forces may be reduced greatly by relatively simple devices. For example, the attraction of the sun on the moon is approximately twice that of the earth. If the earth and the moon were stationary, the sun would quickly pull the moon away from us. But most of the sun's attraction is used up in pulling the moon into approximately the same curvilinear orbit as that of the earth. What is left over is only about 1/100 of the earth's attraction. Consequently, as the first approximation, the moon's orbit around the earth is approximately a Keplerian ellipse. A perturbation as large as 1/100th of the primary acceleration, however, is extremely large, and the accurate determination of the moon's orbit is a very complicated matter.

Two well-known perturbations of satellite orbits due to an equatorial bulge of the central body are 1) regression of the nodes, and 2) advance of the perigee. That is, the nodes gradually move in a direction opposite to that of the orbital motion, while the perigee gradually moves in the same direction as the orbital motion. Perturbations due to drag cause a gradual decrease in the eccentricity and semimajor axis of an orbit. In many cases, the magnitude of these "secular" perturbations can be calculated to a good degree of approximation.

There are several different methods for handling perturbations. In one of them we make no reference whatsoever to auxiliary ellipses, but simply integrate the total acceleration, in order to follow the path. This process, used with numerical integration, is called Cowell's method. It has been used in lunar trajectory

work almost exclusively. A second way to handle perturbations is to calculate from the position and velocity at any point in the actual path the elliptic orbit that would be followed if at that point all perturbations were suddenly to cease. The differences between the actual accelerations and the "two-body accelerations" in this "osculating" ellipse are then integrated to find a correction to a position in the two-body orbit that will give the position in the actual orbit. When numerical integration is used, this method is referred to as Encke's method. It is especially effective when the perturbations are small. After a time, however, the perturbations are likely to build up to such a point that a new osculating reference orbit must be determined from integrated position and velocity.

Instead of making abrupt changes from one reference orbit to another, we can make the changes gradually by the method of variation of parameters. In this method the parameters that define the osculating two-body orbit are allowed to vary progressively so that the osculating orbit will always give the same position and velocity as the two-body orbit. The effect will be to cause one of the osculating ellipses gradually to change until it merges into the other one. The variations of the parameters are determined directly from the perturbations and may be integrated numerically, or, alternatively, by series expansions.

When the perturbations are very large, neither Encke's method nor the method of variation of parameters offers any advantage over Cowell's method, and the last should be used because it requires less calculation. When the perturbations are small, however, and especially when the two-body motion is very rapid, Cowell's method is disadvantageous and may even be incapable of handling the problem.

When perturbations are handled by numerical integration, the process is called special perturbations. When the perturbations are represented by series and integrated term by term, the process is referred to as general perturbations. Today we refer to such series as "Fourier series." Actually the process antedates Fourier by more than two thousand years. In the Ptolemaic system the complex motions of the planets were represented by systems of circles that were equivalent to Fourier series.

It is desirable, at this point, to distinguish clearly between two kinds of trajectory work: "preliminary (or feasibility) trajectories" and "precision trajectories." By preliminary trajectories, we mean qualitative trajectories that are useful in preliminary studies, in which only rough estimates of the amount of fuel, the duration of flight, required guidance tolerances, or similar questions are desired. Precision trajectories, on the other hand, are necessary for accurate space navigation.

The lunar flight trajectory illustrates one of the important distinctions between preliminary and precision work. In preliminary studies of lunar and circumlunar trajectories it is possible to suppose that the moon is moving with uniform velocity in a perfect circle, or that it is a fixed point in a rotating framework. In precision work, however, the rotating framework ceases to be useful. In fact there are no simple mathematical expressions that will represent the moon's position for more than a very brief interval. We must turn to tables of the moon's position, such as those given in the various national ephemerides or almanacs.

Another important consideration in precision orbit work is the following. At the present time refined values of the basic constants are definitely required before an interplanetary ballistic flight to intersect another planet could be successful. This may seem odd, since centuries of astronomical observations have contributed to plotting the elements of the orbits of planets and satellites to six-place accuracy or better, and to determining the mutual perturbations of these orbits caused by the several bodies in the solar system. However, one dominant factor makes these elements unsuitable for successful planet-to-planet navigation. This factor is that while planetary orbital dimensions are known to six-place accuracy or better when expressed in terms of the astronomical unit (the semimajor axis of the earth's orbit), the astronomical unit (au) itself is known to only about one part in 1500 when expressed in terms of meters or feet, the units in which flight design must be made. (Specifically, 1.495×10^8 km $\leqq 1$ au $\leqq 1.496 \times 10^8$ km.)

As a simple example of the effect of this uncertainty in the scale of the solar system on a problem in space navigation, consider the trip from the Earth to Venus along a minimum-energy orbit. Making several simplifying assumptions regarding the eccentricities and inclinations of the orbits of the Earth and Venus, we find that the uncertainty in the semimajor axis of the minimum-energy orbit would be about 172,000 km or 15 diameters of Venus; and this neglects the timing error introduced. One of the first tasks of a flight into interplanetary space should be the measurement of the fundamental astronomical unit of distance in terms of laboratory standards of length. Another basic constant, the gravitational constant, is known to only about three significant figures when expressed in the cgs system or any other laboratory system of units.

Lunar and Interplanetary Flights

A typical preliminary ballistic earth-moon transit trajectory computed by automatic machine is shown in Fig. 4. It is plotted in rotating coordinates so arranged that the earth-moon line appears to stay fixed. This scheme shows the trajectory about as it would appear to an observer standing on the moon. This same trajectory is plotted in inertial coordinates in Fig. 5.

It can be seen in Fig. 5 that the vehicle in this particular transit trajectory will move in a counterclockwise direction in the initial phases of flight; *i.e.*, the advance of vehicle angular position will be in the same direction as the orbital motion of earth and moon. Such

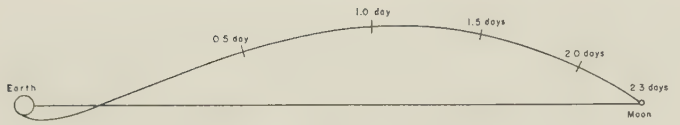


Fig. 4—Moon-rocket trajectory in rotating space.

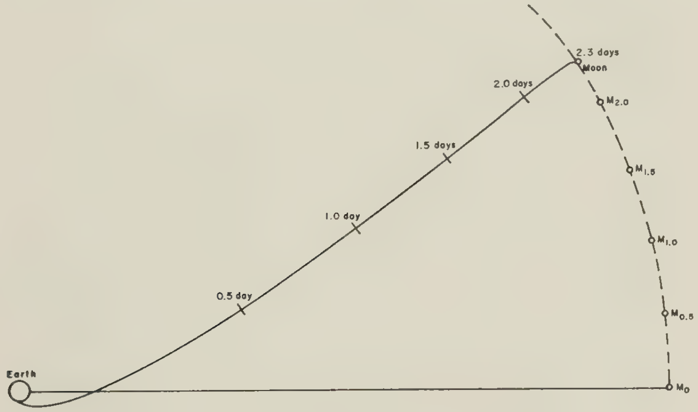


Fig. 5—Moon-rocket trajectory in inertial space.

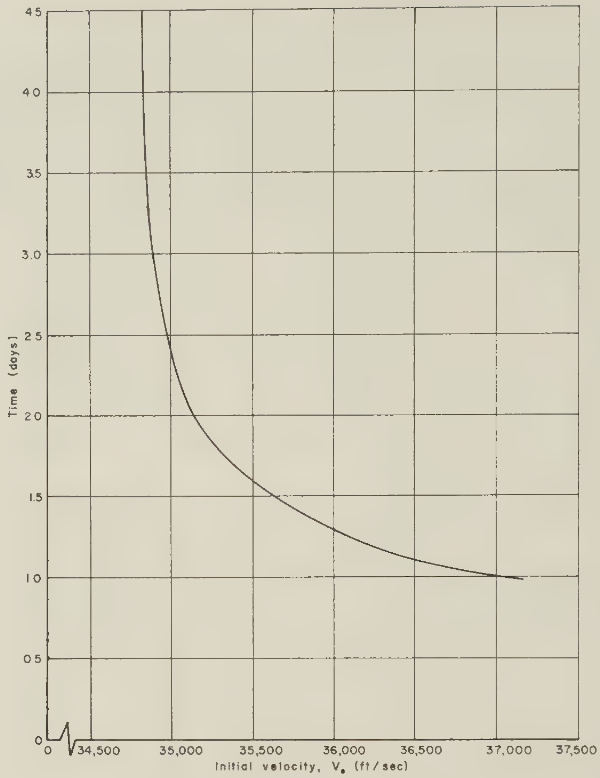


Fig. 6—Transit time from earth to moon.

an orbit is referred to as a direct orbit. An advantage of such an orbit is the fact that one can capitalize on the orbital motion of the earth (in earth-moon space) as well as the rotation of the earth in building up the initial velocity of the vehicle.

In Fig. 5 the attraction of the moon can be seen near the terminal end of the trajectory. The direction of approach has become almost a straight line to the moon's center.

The time required for an earth-moon passage is strongly dependent upon initial velocity. A plot of transit time as a function of initial velocity is shown in Fig. 6. The exact time-vs-velocity curve is, of course, somewhat dependent also upon the direction of projection, but the dependence is relatively slight.

This marked decrease in flight time for a moderate velocity increase in the low-speed regime suggests that the efficiency of some flight missions can be enhanced by sacrificing some payload to increase projection velocity. This would be true, for example, in missions requiring the expenditure of large amounts of electrical energy during transit, or in manned flight where the demands of nutrition and a livable environment grow with flight duration.

A lunar-impact flight consists simply of projection of a vehicle from the earth to crash on the surface of the moon unchecked. Such a flight would typically involve traversal of a trajectory, like that in Figs. 4 and 5. The speed of the body at impact, relative to the moon's surface, will be no less than lunar escape velocity, and typically would be around 10,000 feet per second. It is conceivable that some sort of instrument package could be made to survive such an impact, but the possibilities are only of a speculative sort.

A particularly interesting payload possibility for an impact flight is a source of visible light to signal arrival. It has been estimated that something like 10 pounds of flash power exploded on the dark half of the half-illuminated moon would be observable in a 21-inch reflecting telescope.

The accuracy required in the projection process to produce an impact on the visible side of the moon must be determined by trial and error, *i.e.*, simply by computing a great number of trajectories, noting locations of impacts and miss distances. The values of allowable errors in speed and direction of projection are dependent upon the speed, direction, and position at the initial point in the unpowered trajectory. A coordinate arrangement for defining projection conditions is shown in Fig. 7. Combinations of initial conditions that result in hits passing through the moon's center are shown in Fig. 8. For nominal values

$$V_e = 35{,}000 \text{ feet per second}$$
$$\gamma = 14.2 \text{ degrees}$$
$$\phi = 108 \text{ degrees}$$
$$r = 4300 \text{ statute miles,}$$

marked in Fig. 8, we find that allowable errors in speed or direction for impact on the visible face of the moon are about

$$\delta V = \pm 40 \text{ feet per second}$$
$$\delta\gamma = \pm 0.25 \text{ degree.}$$

The exact band of conditions for impact, around the nominal point selected, is shown in Fig. 9. Generally

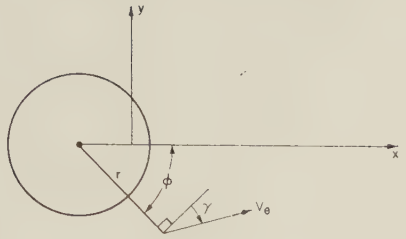


Fig. 7—Parameters used to describe initial conditions.

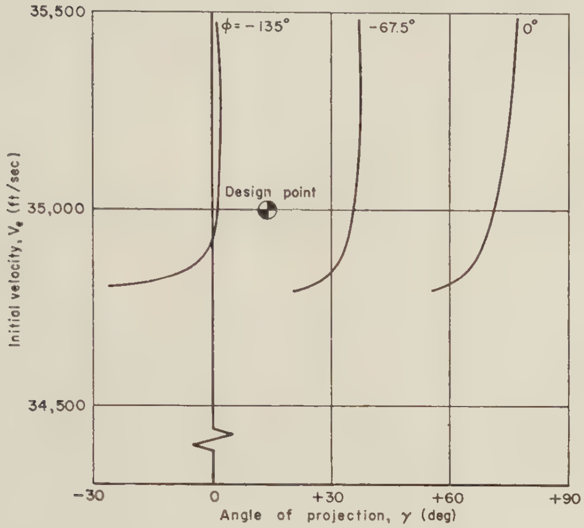


Fig. 8—Combinations of V_e and γ required to hit the moon from various initial positions.

speaking, higher values of V_e lead to larger allowable δV, while smaller values of V_e allow greater values of $\delta\gamma$. Effects of velocity errors are illustrated in Fig. 10.

We must also recognize the existence of another kind of flight tolerance that does not figure in purely terrestrial flight activities—that of launch time. In addition to a fairly close tolerance on the instant of launch, it must be recognized that the calendar dates on which launching is feasible are dependent upon the latitude of the launch site, the range of firing azimuth available, and the inclination of the moon's orbit relative to the earth's equatorial plane. These general observations about launch-time tolerance apply more or less directly to all of the lunar flight types listed.

For most equipments, a nondestructive landing on a solid surface implies an approach to the surface at a rather low speed—a good deal less than 10,000 feet per second. Since the moon has no appreciable atmosphere, deceleration must be accomplished by rocket propulsion in the final phase of approach.

The trajectory requirements for lunar landing are essentially the same as those for impact, except perhaps for some closer specification of accuracy tolerances if a nearly perpendicular hit on the lunar surface is needed to accommodate the particular landing-gear arrangement employed. Landing does, however, involve another extension of the problem beyond the impact case. It introduces a requirement for control of the orien-

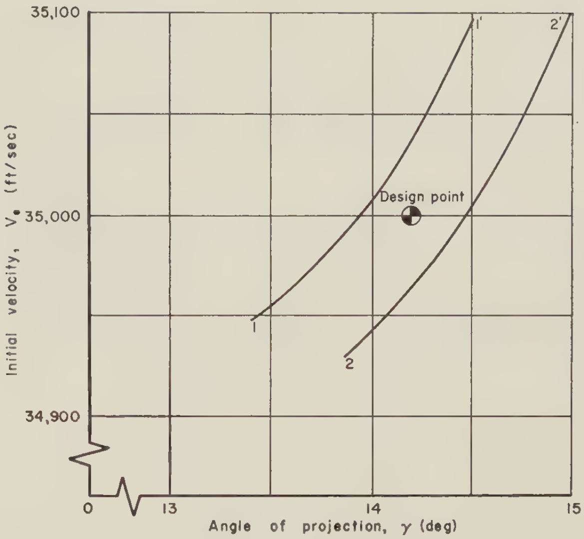


Fig. 9—Hit-band region around design point.

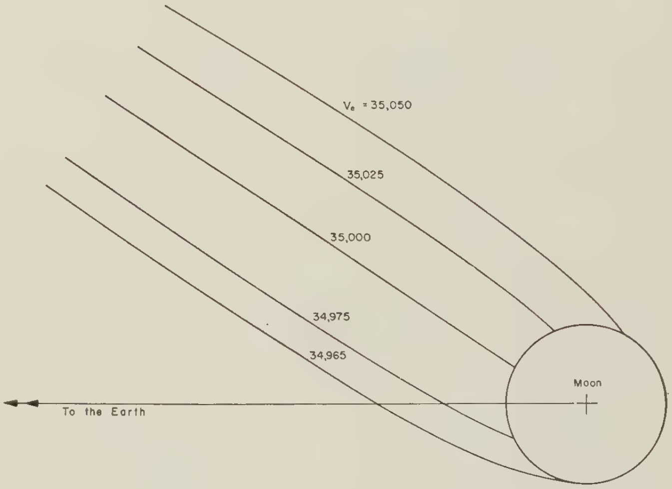


Fig. 10—Effect of varying V_e on hit point.

tation of the vehicle so that the decelerating-rocket thrust is properly aligned relative to the lunar approach velocity.

Another flight mission that requires rocket deceleration at the moon, and, hence, altitude stabilization, is that of establishing an artificial satellite of the moon. For this operation we must proceed along a transit trajectory that misses the moon, to pass by it at an altitude equal to the desired satellite altitude.

The period and orbital velocity of a lunar satellite as a function of orbit altitude is shown in Fig. 11. It is seen that for reasonably close satellites, orbital velocity falls in the vicinity of 5000 feet per second. Since the velocity of the vehicle in its transit trajectory will be of the order of 10,000 feet per second near the moon, it is apparent that a velocity reduction of around 5,000 feet per second is required to set up a lunar satellite.

The projection accuracy required in this operation does not differ markedly from that required to lunar impact. The limiting accuracy requirements are derived from consideration of two possible catastrophies

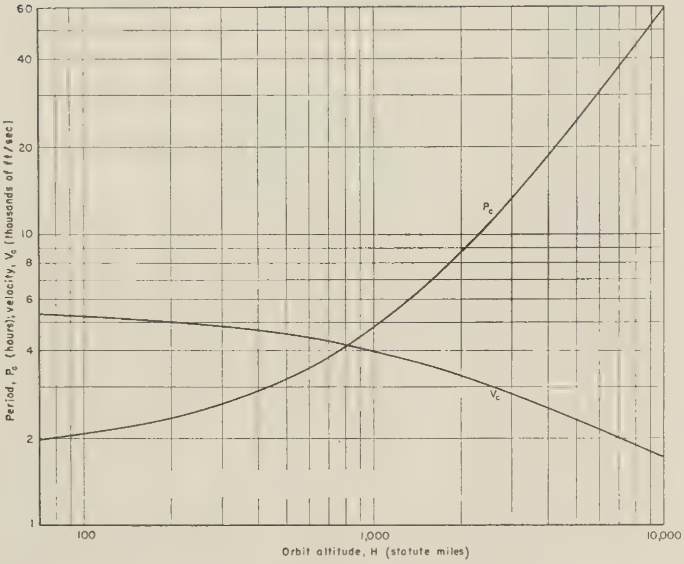


Fig. 11—Period and velocity in circular orbit.

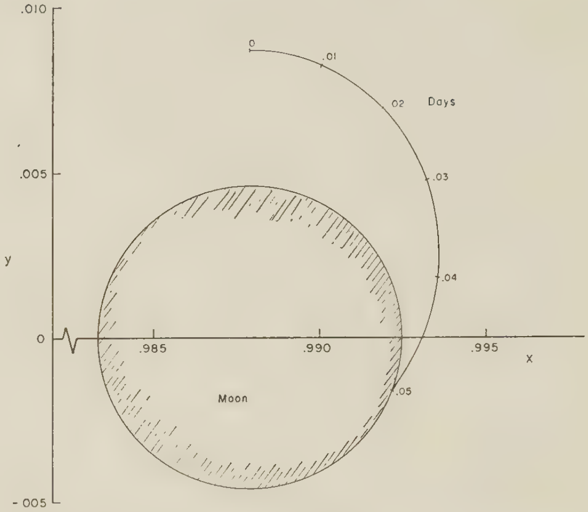


Fig. 12—Collision due to insufficient initial velocity.

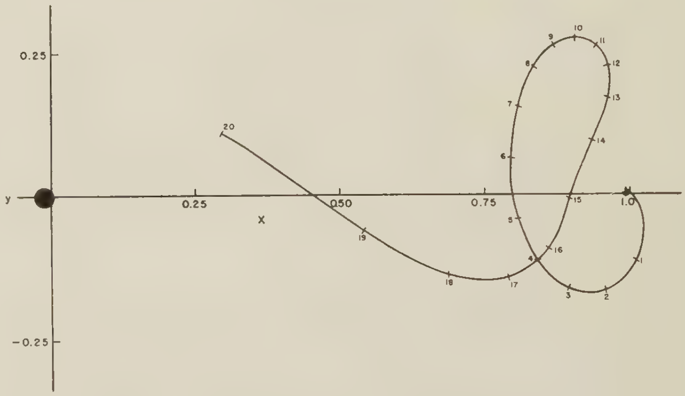


Fig. 13—Recapture of lunar satellite by the earth.

that can occur to the satellite. Too low an initial velocity will cause it to collide with the moon (Fig. 12); too high a velocity will result in recapture by the earth (Fig. 13).

If we wish to make an unpowered flight entirely around the moon and return to the earth, we must stay near the extreme low end of the scale of lunar flight speeds. In fact, we must operate in the region between about 34,800 and 35,100 feet per second only (referred to an initial position 4300 miles from the center of the earth). Substantially higher initial velocities would result in speeds near the moon that are too high to permit sufficient deflection of the trajectory by the moon.

Within the allowable range of initial velocities, the accuracy requirements for circumlunar flight are comparatively modest if all we ask is return to the earth: typically ± 75 feet per second in velocity or ± 5 degrees in direction. These large tolerances are, however, associated with fairly large variations in the distance of closest approach to the moon and in total flight time. A variation of 10 feet per second in initial velocity would change the distance of closest approach by about 1000 miles and the total flight time by about 25 hours. Because of this sensitivity of flight time to initial velocity, the velocity would have to be controlled to within about ± 0.5 foot per second if a returning circumlunar vehicle were to be recovered within the continental United States. These values of sensitivity apply to a trajectory with an initial velocity of about 34,900 feet per second which passes the moon at a nearest approach distance of about 4000 miles. The sensitivities for other trajectories could differ from these by as much as an order of magnitude depending upon the exact values of the initial conditions.

There are five special points in earth-moon space, called "libration centers," at which a vehicle might "float at anchor" as a sort of space buoy. The arrangement of these points in the (x, y) plane is shown in Fig. 14. Approximate solutions to the equations of motion can be developed in the neighborhoods of these centers of libration.

We find from this solution that the motion near the straight-line centers of libration (I, II, and III) is unstable; because of the presence of the hyperbolic functions, a particle initially near a center of libration will eventually move indefinitely far away.

For the equilateral-triangle points only oscillatory terms appear in the solution to the equations of motion, so it would seem that we could establish space buoys at the triangle points that would stay at anchor in earth-moon space for an indefinite period, until displaced by external disturbances.

In treating lunar flight, we have been concerned with a space environment dominated by the fields of two massive bodies—the earth and the moon—revolving in circles about their common center of mass. When we consider interplanetary flight, the main features of the problem are determined by a similar kind of flight environment. The difference is that the interplanetary flight has more major phases.

Let us run through these phases in a flight, say, from earth to Mars. The first phase takes place in earth-moon space. This phase soon blends into the second

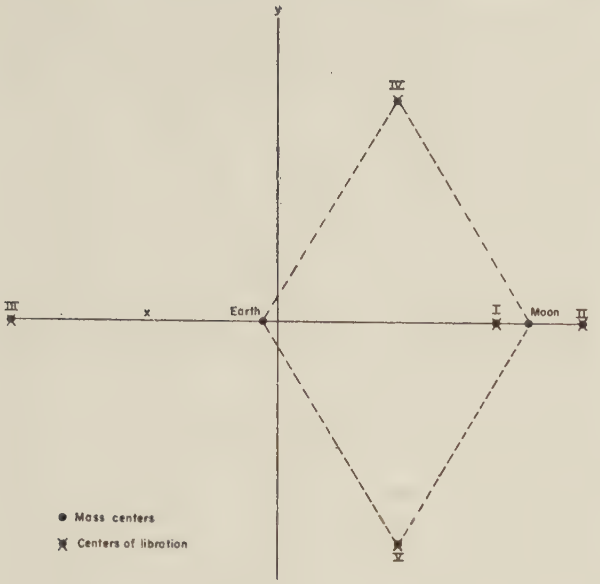


Fig. 14—Relative positions of libration centers.

phase, where the main sources of influence are the earth and sun. At a distance of a few million miles from the earth, the third phase begins, in which the sole influence of substantial consequence is due to the sun. As we approach Mars, we enter the fourth flight phase, where the bodies of chief concern are Mars and the sun. In the terminal, or fifth phase very near Mars, only the field of Mars itself is important.

The computation of an interplanetary flight trajectory is very complex, because of the multiplicity of flight phases with the attendant requirements for changing reference frames, equations of motion, accuracy scales, etc. However, the major characteristics of an interplanetary trajectory can be summarized as follows. The initial leg of the trajectory (phase one) approximates a hyperbola with focus at the earth's center; this leg (through phase two) blends into a large ellipse with focus at the sun's center (phase three); near the end (through phase four) this ellipse blends into a hyperbola with focus at the center of the target planet (phase five).

Landing on Mercury would be similar to landing on the moon; there is no atmosphere, so deceleration must be accomplished by rocket. Landings on Venus, Mars, or the earth can make use of aerodynamic drag for deceleration. Landings in the usual sense are not likely on the other planets, since they do not have (or probably do not have) clearly defined solid surfaces.

Establishment of an artificial satellite of another planet involves the same possible sources of failure as establishment of a lunar satellite—too little velocity will result in collision with the planet; too much will lead to capture by the sun. A round trip around, say, Mars with subsequent return to the earth is possible by proper trajectory arrangements.

Libration centers in interplanetary space are produced by the fields of the sun and a planet, just as they are produced in earth-moon space by the fields of the earth and moon. Thus we should also be able to establish interplanetary space buoys. In fact such buoys already exist in natural form as the Trojan asteroids (see below) at the equilateral-triangle points relative to the sun and Jupiter.

For all of these interplanetary missions the guidance accuracy requirements are far more stringent than for analogous lunar missions. Representative velocity tolerances are on the order of 0.1 foot per second.

Another type of interplanetary mission is that of establishing an artificial asteroid (artificial solar satellite).

The Space Environment

One of the most important aspects of the space environment deals with the material content of space. Let us first consider bodies in the range from cosmic dust to chunks of rock (*i.e.*, say 20 microns to a few meters) commonly called meteoroids. Fig. 15, based upon the observational and theoretical results of the Harvard Meteor Program, gives the mass and size of meteoric particles as functions of the visual magnitude. Fig. 16 indicates the number of such meteoroids striking the earth per day, and the number striking a 3-meter sphere in the neighborhood of the earth per day.

It is estimated by Whipple that a meteoroid of magnitude 17, moving with a velocity of 18 km/second, of which about two per day will strike a 3-meter sphere, will penetrate an aluminum skin of 0.01 cm; whereas a meteoroid of magnitude 5, one of which will strike the sphere every hundred years, would penetrate 4.5 cm. of aluminum. About every 50 days a particle capable of penetrating 0.5 cm of aluminum would hit the sphere.

But the probability of striking meteoroids depends upon where the vehicle is in space. Fig. 16 applies to the immediate neighborhoood of the earth. At greater distances good data are lacking. What is known, however, is that 1) the smallest dust particles (micrometeoroids) are concentrated in the ecliptic or plane of the earth's orbit, and 2) most meteoritic material is cometary refuse and is consequently largely distributed along the orbits of comets.

Let us review some of the evidence for the ecliptic concentration of cosmic dust. After evening twilight, especially near March 21 in northern latitudes, a faint tapered band of light can be seen extending up from the horizon centered along the ecliptic. This band of light, which can be photoelectrically traced through the complete night sky, is called the zodiacal light. The color of the zodiacal light is nearly the same as that of the sun, but shows approximately 20 per cent polarization. These observational facts suggest that the zodiacal light is caused for the most part by sunlight scattered from small dust or meteoroidal particles at least 20 microns in diameter. Since light scattered by free electrons is strongly polarized, it is probable that free electrons represent a fraction of the particles present. This

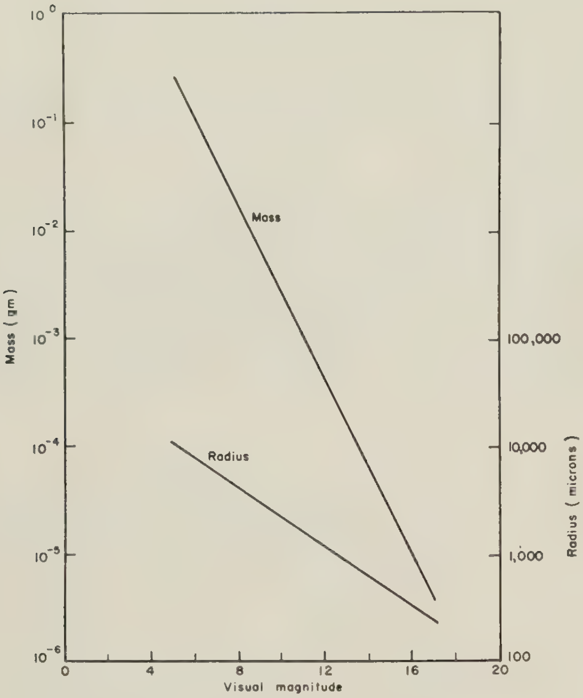


Fig. 15—Meteor brightness vs size.

is also substantiated by the fact that the total light present seems to vary with solar activity, being least when ionizing radiations from the sun are at a minimum. However, since scattering by gas atoms and molecules alters the color of the light, it must be concluded that the zodiacal particles (except for the free electrons) are much larger than molecules.

It has been suggested that the zodiacal light is an extension of the outer solar corona. This idea is reinforced by the fact that the corona has a color and continuous spectrum agreeing with the zodiacal light. But most interesting is the comparison of the brightnesses, as shown in Fig. 17.

This layer of small meteoroidal particles must extend from the sun well beyond the orbit of the earth, being concentrated toward the ecliptic or fundamental plane of the solar system.

The major concentration of the smallest meteoric material (producing no visual effects when striking the earth) is in the ecliptic, but other concentrations are intimately associated with comets and other bodies. The visible meteors, or shooting stars, are of two types—those associated with showers and those which are sporadic. The shower meteors are of cometary origin; the sporadics are proably traceable to asteroids.

Let us review a few facts concerning comets and meteor showers. No accurate masses of comets have been determined, since they are not massive enough to exert any measurable perturbative forces on other bodies. But it is estimated that typical masses are of the order of 10^{12} tons (earth = approximately 10^{21} tons), and the densities are such that in a thousand cubic miles of a comet's tail there is less matter than in a cubic inch of air.

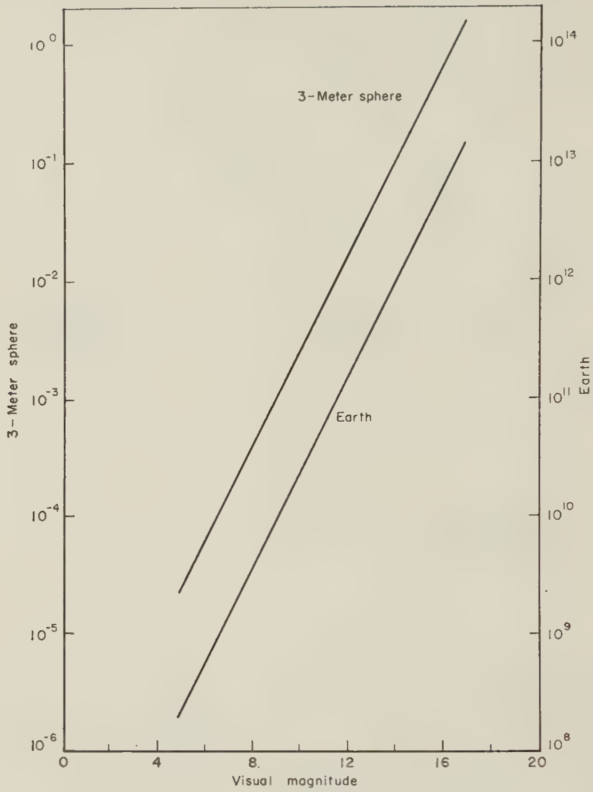


Fig. 16—Meteor impacts per day.

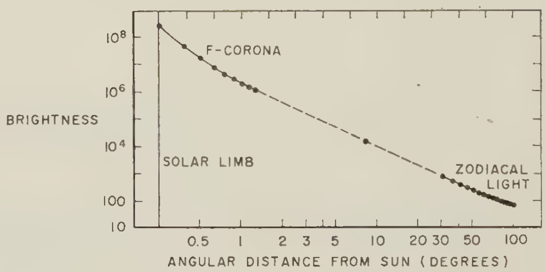


Fig. 17—Change of brightness of the sun's outer corona and the zodiacal light with distance from the sun.

In 1949 Whipple hypothesized a comet model which satisfactorily explains a great many observed facts about comets. Whipple holds that a comet's nucleus is a cosmic iceberg, a porous mass of solidified gases or ice plus some solid particles. The substances present are largely water ice, ammonia, and methane with some carbon dioxide and cyanagen.

But what is of special interest is that on each trip near the sun, the comet is partially disintegrated and leaves a "wake" of small solid particles and ices. So the regions of space where an astronaut is likely to find higher than average densities of meteoric material are along the orbit of comets, either "live" comets or old disintegrated comets.

Whenever the earth passes through one of these cometary wakes, a meteor shower results. Hundreds of shooting stars are observed to emerge from a small area of the sky called the radiant, the direction being determined by the orbit of the comet wake in space. In

general these small, solid particles or bits of ice a few microns in size, which cause meteor showers, will not cause penetrative disasters to a space vehicle, though they may in time cause considerable skin attrition. It is the sporadic meteoroids that are likely to cause sudden trouble in space flight. These bodies are most probably fragments of asteroids which have resulted from collisions. Like comets, none seems to have a definitively hyperbolic orbit. However, these sporadic meteoroids may be quite sizable, form fireballs, and frequently strike the earth. They range from a few grams up to thousands of tons, like the large meteorites (or even small asteroids) which caused such craters as the Barringer Meteor Crater in Arizona.

Let us now turn briefly to some facts concerning the minor planets or asteroids themselves. Since the discovery of the first asteroid on January 1, 1801, the orbits of more than 1500 of these bodies have been determined. However, their total number must run into the hundreds of thousands; it has been estimated that there are 80,000 brighter than the 19th magnitude alone. Most of the asteroids follow orbits which lie between the orbits of Mars and Jupiter, occupying a place in the solar system where Bode's Law has predicted a major planet which does not exist. (Some asteroids depart considerably from the mean orbits.) One family of asteroids is of special interest. It occupies the equilateral libration points in Jupiter's orbit (Fig. 18). These asteroids—

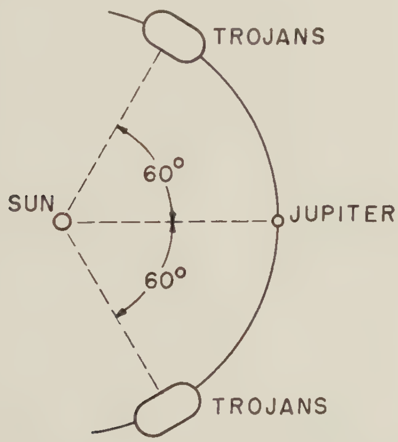


Fig. 18—The Trojan asteroids.

known as the Trojans—number about 12, some leading Jupiter, some following. Searches have been made for possible Trojan-type asteroids associated with the equilateral libration points in the orbits of other planets, but none has been found.

Orbits whose periods are exact fractions of Jupiter's period are called resonant orbits. The effect of perturbations on these resonant orbits is to render them unstable and force the asteroids into other orbits, a fact which might be of interest to astronauts; similar effects would operate on earth satellites whose periods were exact fractions of the lunar period. Thus, if a satellite were placed on an orbit with a period of, say, exactly $\frac{1}{4}$ month, it would soon move into some other orbit.

In recent years high-powered, wide-field photographic telescopes have recorded thousands of faint new asteroids, some of them on orbits which bring them close to the earth; in 1937 an asteroid swept within 800,000 km of the earth, or roughly twice the moon's distance. Orbits are now known for at least ten such objects which come within the earth's orbit. Undoubtedly there are scores more; and over a period of hundreds of thousands of years collisions with the earth must occur.

The largest asteroid (and the first discovered) is Ceres, with a diameter of 730 km. The sizes range on down to a few kilometers. Assuming that the ratio of reflecting power to size is the same for small asteroids as for large ones, we have

Absolute magnitude	5.0	10.0	15.0	20.0
Diameter (km)	270	27	2.7	0.27

Since the number of bodies increases by a factor of 2.7 with each magnitude, there are probably 100,000 asteroids with diameters in excess of 250 meters. It is estimated that all the asteroids together would make up a spherical body about 1000 km in diameter with a mass less than one-thousandth the earth's mass.

Interplanetary space also contains molecular, atomic, and subatomic particles and radiation of various kinds.

In the exospheric region of the terrestrial atmosphere great numbers of nitrogen, oxygen, and other particles are freely orbiting as a highly tenuous atmosphere. At higher levels of the exosphere lighter gases, such as hydrogen and helium, may eventually assume an increasingly important contribution to the density and composition. The proportion of ionized particles to neutral atoms will increase to values such as one in five, and more, at greater distances from the earth, because there will be few collisions between the very highly ionized positive ions and negative electrons; the probability for neutralization of the electric charges by recombination will be very small. At very high levels or beyond the atmosphere protons and electrons will dominate, together with some neutral hydrogen atoms. The electron density at the base of the exosphere can be taken as $3 \times 10^7/\text{cm}^3$. Indirect data on the solar corona and zodiacal light suggest that the region between the earth and moon has an electron density of the order of $10^3/\text{cm}^3$.

The lunar gaseous atmosphere may consist mainly of argon, together with carbon dioxide and sulphur dioxide and some water vapor, but its true composition and density are as yet uncertain. It is also possible that the moon has an ionosphere with electron densities of the order of $10^5/\text{cm}^3$; some estimates go as high as $10^{10}/\text{cm}^3$.

According to the Chapman-Ferraro theory of magnetic storms, corpuscular streams of electrons and protons are emitted from active regions on the sun, and proceed earthwards to cause magnetic storms and auroras. These particles, according to this theory and several others related to it, travel to the earth in about a day, so that the velocity may be about 10^8 cm/second.

Solar particles moving with a velocity of about $\frac{1}{3}$ that of light have been noted to leave the sun in areas of solar flares. (Methods of radio astronomy have been used in these studies.) During solar flares on about six occasions since the early 1940's, marked increases in cosmic rays have occurred over a period of hours to almost a day. On February 23, 1956, an increase of 90 per cent or so in cosmic rays detected at the ground appeared in high latitudes of the earth. This means that the particles had an energy in excess of 18 billion electron volts (bev.). Effects persisted over a period of 16 hours or more in cosmic rays, and for several days high absorption of radio waves ensued. The potential extreme radiation hazard like that of February 23, 1956, apparently does not occur very often; this and some other large increases during flares have appeared only about once every three years.

A weak galactic background of radio noise from a few to thousands of megacycles exists. It is believed to arise from electrons spiraling in magnetic fields of active areas in distant galaxies. Some localized areas radiate very intensely, as in the region of the Crab nebula.

The sun also emits a radio noise background. Since a black body at some thousands of degrees generates electromagnetic waves whose intensity varies with wavelength, there must be emission at radio frequencies as well as in the ultraviolet, the visible spectrum, and the infrared. Also, during solar flares the sun emits short bursts of radiation up to 1000 times as great as its steady background radiation.

The earth's magnetic field in space is much like that of a short magnet at the earth's center, the magnet being so directed that its north pole will lie in the general direction of the geographic poles. The central axis of this magnet intersects the earth's surface at the point 78.6° north latitude and 289.9° east longitude, called the geomagnetic north pole. The magnetic moment of this magnet, taken as a very short magnet, or dipole, was 8.06×10^{25} centimeter-gram-second units in 1922.

At points beyond the atmosphere at distance r from the earth's center, the magnetic field falls off rather nearly as the cube of the distance. The electric currents flowing in the atmosphere, believed mostly transients, add to the main magnetic field. The main field includes also some higher-order terms required in precise calculations of the field in space.

The surface magnetic field of the sun is not much larger than that of the earth, except within sunspots. The field may vary somewhat with time, and a magnetic moment of the sun is difficult to assign. In the case of sunspots, there are usually local north and south magnetic poles in the sunspot groups. Magnetic moments may be as great as or greater than values 10^6 times that of the whole earth when spot diameters reach 50,000–60,000 km, with magnetic fields of the order of 5000 oersteds.

Such sunspot fields could therefore extend well beyond Mercury and almost to the planet Venus with fairly readily measurable intensity, if it were not for the fact that the solar corona is a very good electrical conductor; as a consequence, electromagnetic induction tends to nullify systematic features of the changing sunspot fields, except at points close to the sun. However, it is expected that portions of the actual magnetic fields of sunspots are carried by material within moving prominences or the streaming corona to the neighborhood of the moon and earth with measurable intensity of magnetic field. Hence, the sunspot fields are expected to exist in fragmentary and badly organized form within the solar system.

The relative effect of the hot solar corona on space vehicles will be negligible for a lunar flight. The heat flux from the solar corona per square centimeter of area will be kinetic energy of motion; but the hot particles, though very energetic, would be too few in number to heat up a metal surface appreciably. The number of protons and hydrogen atoms should be of the order of 10^3 to $10^4/\text{cm}^3$, so that the energy flux would be only 10^{-7} times the maximum solar radiation flux of nearly two horsepower per square meter.

Experimentation in Space

The use of space flight for scientific experimentation will greatly add to the stock of scientific knowledge, and, of course, such experimentation is also necessary for the successful fulfillment of many space-flight missions. Looking beyond the IGY program, we are able to foresee such useful experiments as, for example, the refinement of basic constants (planetary masses, gravitational constant, dimensions of the solar system). For these purposes, artificial asteroids (satellites of the sun) and planetary satellites, perhaps with transponding equipment for accurate range and range-rate measurement, are one possibility.

There are several other uses for artificial asteroids. When tracking techniques at the distance of Venus, for example, have been perfected, an asteroid on an orbit making a close encounter can be used with perturbation theory as a test particle for refining the mass of the planet. Asteroids carrying suitable instruments can study the effects of solar particle radiation in regions of space remote from the perturbing effects of the earth's magnetic field. If instrument-bearing asteroids could be placed in the earth's equilateral-triangle libration points, observations of the directional properties of solar flares and spots could be made. Asteroids with suitable impact counters could map the distribution of meteor streams in all parts of space to determine optimum courses for later interplanetary vehicles.

Another sort of interplanetary vehicle would be an artificial satellite of another planet. It should be possible to learn a good deal about planets and their atmospheres from satellite observing stations. A logical prelude to actually landing on a planet (though probably not a necessity for Mars or Venus) would be observation of the behavior of an instrumented "reentry body" as it

plunged into the planet's atmosphere. From a knowledge of its approach trajectory and a time history of altitude, deceleration, and vehicle surface heating, the atmospheric data necessary to design subsequent entry vehicles could probably be determined.

What is the present state of knowledge concerning these neighbors of the earth? First, Venus. Actually very little is known about Venus. Its rotation period is very uncertain; since it has no satellites, its mass is known to only 3 per cent; and since it is covered with opaque clouds, nothing concerning its surface is known. Even the chemical composition of the Venusian atmosphere is controversial. Large amounts of carbon dioxide have been observed but no evidence of water or oxygen. Some believe that Venus is a dry, dusty planet covered with an opaque dust cloud. Others believe that Venus is one vast ocean, and that water has not been detected in the atmosphere because it is always in the form of ice. Still others believe that the clouds are formaldehyde and that Venus is covered with plastics. These hypotheses are not idle speculations but are consistent with the observations. It is the difficulty of getting suitable observations that leaves the conditions on Venus so uncertain.

Bolometric observations of Venus suggest some rotation. Richardson has recently concluded that Venus has a rotation period of from 8 to 46 days, with a probability of 0.5 of being correct. He claims that 14 days retrograde is the best mean value. The axis, as ascertained from cloud markings, is tilted from the plane of the orbit somewhere between 14 and 32 degrees (not so different in this respect from the earth and Mars). The fact that no equatorial bulge has ever been observed and that radio measurements showing a 13 day fluctuation have been observed, strengthen the case for Richardson's 2-week Venusian day. That oxygen has bever been observed may be traced to the fact that all observations are restricted to the upper parts of the atmosphere, where oxygen is probably dissociated, as in the earth's atmosphere.

All of these statements add up to the probability that Venus will be a "surprise planet" when visited by pioneer astronauts. Nothing is definitively known which precludes the existence of conditions favorable to life. And at least one prominent astronomer feels that Venus will be the planet on which we are most likely to find life.

As to the earth's other neighbor, Mars, a great deal more is known. Mars rotates on its axis in 24 h 37 m, or essentially one earth day. Its axis is inclined to the orbital plane by the same amount as the earth's, and seasonal effects similar to those of the earth are observed.

The conditions on the surface of Mars are very similar, with regard to temperature and pressure, to conditions on the earth 11 miles above the surface in the stratosphere. Although human life could not survive without extensive local environmental modifications, the possibility of a self-sustaining colony is not ruled out.

But bleak and desert-like as Mars appears to be, with no oxygen and very little, if any, water, there is good evidence (derived from observation of the Martian dark areas and seasonal color changes) that some indigenous life forms may exist.

The canal controversy is still unsettled and probably will remain so until Mars can be adequately observed from a position free from the blurring motion of the earth's atmosphere.

Already, through the study of cloud movements and temperature distributions on Mars, knowledge is being gained which is useful in the analysis of the earth's atmosphere.

Each planet, regarded as a scientific laboratory, offers unlimited possibilities for studying physics, geology, meteorology, chemistry, and even life science. The scientific dividends from the exploration of space should, in not too long a period, repay the whole cost many times over.

It has been noted that nearly all the physical attributes of the exosphere, solar corona, and lunar atmosphere are so ill-known that it is highly desirable to conduct the basic research needed to remove the dearth of real knowledge.

Among the physical experiments that might be conducted (in addition to those mentioned above) are the following:

1) Measurements of the composition, density, and temperature of matter along the path of a lunar flight, and on the moon.
2) Measurement of X-ray and ultraviolet radiation along the flight path. Some attention to infrared radiation also seems indicated.
3) Measurement of the spectrum and intensity of radiation at radio frequencies from space, from the the sun, and from sunspots.
4) Measurement of the geomagnetic field at various distances and of possible magnetic fields accompanying auroral streamers and ring currents. Turbulent magnetic fields within the solar corona should also be measured. The lunar magnetic field should be ascertained, right down to the lunar surface.
5) Cosmic ray observations with counters along the flight path. On the moon, directional experiments will be useful in the study of cosmic radiation from the sun or special sources requiring precise location.
6) Precise measurements of the lunar mass and gravitational field.
7) A mass spectrograph on the moon to identify gases, such as argon, xenon, krypton, carbon dioxide, sulphur dioxide, and water vapor.
8) Seismic observations, with or without explosions, to provide information on the lunar interior and composition.
9) Measurement of radioactivity at various depths within the moon.

Space Communications*

PETER SWERLING†

Summary—Preliminary design parameters for space communications can be estimated by standard application of the radiation range equation and of communication-theoretic equations relating channel capacity to bandwidth and signal-to-noise ratio. An example is given for a hypothetical situation involving communication between a space vehicle and a receiver located on the earth's surface, for various communication ranges.

Preliminary estimates of this type also serve to indicate areas of research and development which will be of importance in space communications. Some of the areas briefly mentioned are: components (vehicle and surface antennas, sensitive receivers, highly reliable circuit components, and electrical power sources); signal storage, encoding, and processing techniques; tracking and acquisition techniques; and studies of the physical environment (extraterrestrial noise, component environment, and electromagnetic propagation environment in space).

Introduction

SPACE communications can be defined as the design and utilization of communication equipment for space-flight applications. Much has already been accomplished in the field of space communications, *e.g.*:

1) The minitrack[1] and microlock[2] systems and other communication equipment associated with our own satellite and lunar probe programs.
2) The communication systems associated with the Sputniks.
3) The communication design studies associated with military uses of satellites.

It is probable that communication techniques in space will be, at least for a long time, an evolutionary development of those techniques already in use in current space-flight programs and of closely related techniques developed in fields such as air defense surveillance and radio astronomy.

Aside from the purely operational definition given above, one can speculate as to whether the subject of space communications will become a distinct branch of communications in the theoretical sense. In what way, and even whether, this will happen is not at present readily foreseeable. Basically the same equations will govern the propagation of electromagnetic energy and the transmission of information in space as on the earth.

We may, of course, expect certain practical differences between the conditions of space-flight communications and those of terrestrial communications. Among these are the following:

1) The space environment. This refers both to the physical environment in which equipment will operate and to the propagation environment associated with interplanetary space and with the various atmospheres and ionospheres of extraterrestrial bodies.
2) Very large communications ranges.
3) Severe size and weight limitations in space vehicles and severe reliability requirements for lengthy unattended operation.

One can combine what is known about the conditions under which space communications will have to operate with the basic equations of communication theory to predict the general lines of research and development which will be useful in accomplishing the communication tasks required by space-flight programs. The discovery of any really novel effect such as an unexpected propagation effect in one of the planetary atmospheres, or the practicability of using some other source than electromagnetic energy in communications must await further developments.

Illustrative Computation of Power Requirements

Communications engineers in a fairly large number of places (see Lehan[3] for an example) have already investigated the basic requirements in terms of radiated power, antenna gains, etc., for a variety of space-communications tasks. The starting point for these investigations is invariably 1) the radiation range equation, and 2) communication-theory equations relating information-transmitting capacity to bandwidth and signal-to-noise ratio.

These equations are useful not only because they enable one to calculate basic requirements, but also simply because they list many of the parameters which are of greatest importance to communications and thus provide a useful framework for discussion.

The (one-way) radiation range equation is

$$S_r = \frac{P_t G_t A_r}{4\pi R^2 L} \qquad (1)$$

where

S_r = received signal power,
P_t = radiated power,
G_t = transmitting antenna gain,
A_r = effective receiving area of the receiving antenna,
R = communication range,
L = losses.

* Manuscript received by the PGMIL, August 20, 1958.
† The RAND Corp., Santa Monica, Calif.
[1] R. L. Easton, "The Mark II Minitrack System," U. S. Naval Res. Lab., Washington, D. C., Rep. No. 5035, Project Vanguard Rep. No. 21; Minitrack Rep. no. 2; September, 1957.
[2] H. L. Richter, W. F. Sampson, and R. Stevens, "Microlock; a minimum weight radio instrumentation system for a satellite," in "Vistas in Astronautics," Proc. of the First Annual AF Office of Sci. Res. Astronautics Symp., Pergamon Press, New York, N. Y., 1958.
[3] F. W. Lehan, "Space communication—implementation problems," Lecture 10B, lectures on space technology, Univ. of Calif., Berkeley; 1958.

The factor L in this context could refer to such losses as absorption losses, polarization losses, or losses due to imperfect antenna pointing.

One standard formula for the communication capacity C of a channel is[4]

$$C = W \log_2 \left(1 + \frac{\overline{S}_r}{N_0 W}\right) \tag{2}$$

where

$\overline{S}_r$ = average received signal,
W = channel bandwidth,
N_0 = noise power density,
C = maximum or ideal rate of transmission in bits per unit time.[5]

The above formula applies to the case of a signal received in additive Gaussian noise with uniform noise power density N_0 over the bandwidth W. Several of the most important sources of noise are of this form (or very nearly so). For example, if the communication channel bandwidth is narrow compared with the center frequency (which is usually the case), then internal receiver noise and solar and cosmic noise are approximately of this form.

For present-day receivers, the value of N_0 is roughly 10^{-14} to 10^{-13} watt per megacycle. The noise power density tends to increase with the center frequency in present receivers. Future ultrasensitive receivers such as masers,[6] which are well along the development road at present, may reduce internal noise power density by a factor of 100 or more.

The intensity of solar and cosmic noise has already been extensively investigated by radio astronomers.[7] The noise power densities of these noise sources tend to decrease with increasing frequency. For an omnidirectional receiving antenna located approximately in the earth's orbit, the cosmic noise at, say, 250 mc would be perhaps a factor of ten or so less than present-day receiver noise, while a "quiet" sun would be still less noisy. During solar "noise storms," however, the intensity of solar noise may increase by factors of ten to 1000.

If the receiving antenna is directive, and if it happens to be pointed at a strong noise source such as the sun, extraterrestrial noise could be the predominant noise source. However, the illustrative figures cited above indicate that, with today's receivers, receiver noise will ordinarily exceed noise from extraterrestrial sources. However, if maser receivers actually succeed in reducing receiver noise by a factor of, say, 100, extraterrestrial noise (or perhaps man-made interference) will become the limiting factor, so that over-all noise intensity would not be reduced by so large a factor.

Another useful relation which can be derived from (2) is

$$C < \frac{1.4\overline{S}_r}{N_0}. \tag{3}$$

Table I illustrates how (1) and (3) may be used in preliminary estimation of the requirements of various space communication tasks. The table gives ideal minimum average radiated power necessary to achieve the stated transmission rate C at the stated range R. (For those who prefer to think in terms of signal-to-noise ratio and bandwidth, the table entries also represent the radiated power required to achieve a unity signal-to-noise ratio in a channel having bandwidth in cycles per second equal to 0.7 times the stated number of bits per second in the column headings.)

The parameter values for which the table entries were computed are intended purely for illustrative purposes, and are supposed to be reasonably realistic values for transmission from a space vehicle and reception on the earth's surface. They are:

$A_r = 10^4$ meters^2,
$G_t = 1.00$,
$L = 10$,
$N_0 = 2 \times 10^{-14}$ watt per megacycle.

The 10,000-square-meter receiving antenna is larger than the largest present-day antennas by a factor of perhaps ten. The noise power density is representative of receiver noise in current receivers.

An omnidirectional (unity-gain) transmitting antenna is assumed for the space vehicle; this would impose no stringent attitude-stabilization requirements on the transmitting antenna.

TABLE I

IDEAL MINIMUM VALUES OF AVERAGE TRANSMITTED POWER FOR GIVEN RANGE AND REQUIRED CHANNEL CAPACITY

Range	Minimum required values of average transmitted power		
	C = 10 bits/sec	10^3 bits/sec	10^6 bits/sec
5×10^4 km (distant earth satellite)	4×10^{-6} watt	4×10^{-4} watt	0.4 watt
5×10^5 km (lunar distance)	4×10^{-4} watt	4×10^{-2} watt	40 watts
10^7 km (near asteroid)	0.2 watt	20 watts	2×10^4 watts
10^8 km (Mars at close approach)	20 watts	2×10^3 watts	2×10^6 watts

The required power levels in a table of this type depend sensitively upon the particular parameter values assumed, so that it would be misleading to conclude on the basis of a single example that power requirements have been definitely ascertained for a given communication task. Even when the power requirements are

[4] C. E. Shannon, and W. Weaver, "The Mathematical Theory of Communication," University of Illinois Press, Urbana, Ill.; 1949.
[5] Bits per second if W is expressed in cycles per second.
[6] W. H. Culver, "The maser; a molecular amplifier for microwave radiation," *Science*, vol. 126; October, 1957.
[7] PROC. IRE, vol. 46; January, 1958. (Radio astronomy issue.)

clearly very small, it does not necessarily follow that the communication problem is solved; the main difficulty may lie somewhere else than in power requirements.

There are several factors in practical situations which would tend to increase the actual power consumption to greater values than those calculated on the basis of (1) and (3). Among these are the following:

1) The values shown represent ideal minimum power, that is, for ideal or near-ideal information coding. Practical channels do not ordinarily transmit at the ideal rate. The factor by which power must be increased to compensate for lack of ideal coding depends strongly upon the particular situation, but typically might range from five to 100.

2) The values shown represent radiated power; power consumption will always be greater than this, because of lack of perfect efficiency. Consumed power might exceed radiated power by factors of from two to ten (and higher in some situations).

On the other hand, there are techniques which might be used to improve the parameter values assumed for Table I, thus lessening power requirements. Among these are:

1) Use of ultrasensitive receivers; these might reduce the over-all noise level by factors of from ten to 100 (depending upon the external noise intensity).

2) Use of directive vehicle antennas; this could reduce power requirements by factors of 100 or more, as compared with the use of an omnidirectional vehicle antenna, provided that the resulting antenna-beam stabilization and tracking problems could be solved. There would be a trade-off here between the power saving on the one hand, and the added weight and complexity necessary for antenna-beam stabilization on the other.

3) Storage and retransmission of data at a slower rate; this would reduce the required number of bits per second. This would involve a trade-off between the power saving and the weight and complexity of data storage equipment. Also, it could be done only if the information source were intermittent. On the other hand, the requirement for data storage and retransmission may often be imposed by other factors, such as line-of-sight considerations.

Sometimes it is required to send a certain total amount of information rather than to send information at a given rate. In this case the requirement imposed is one of energy rather than of power. If T represents the total time of transmission, one may obtain from (3) the relation

$$\overline{S}_r T < 0.7 N_0 (TC). \tag{4}$$

Here TC represents the total number of bits of information, and $\overline{S}_r T$ the total received energy.

As an example, suppose one desired to transmit a photograph of the surface of Mars. Let us say, as a purely illustrative example, that the area to be photographed is 10,000 square miles; the resolvable elements are required to be one-tenth mile squares; and there are to be eight distinguishable gray levels. Then (neglecting redundancy in the photographed surface) the required number of bits to be transmitted would be 3×10^6. Using the same parameter values as in Table I, for $R = 10^8$ km we get a minimum radiated energy requirement of 5×10^6 watt-seconds. If the total transmission time were 5×10^4 seconds (14 hours), the minimum average radiated power would be 100 watts.

Indicated Useful Areas of Research and Development

The following are some areas of research and development which, in the light of the considerations above, take on importance in the achievement of communications tasks associated with space flight.

Electrical Energy and Power Sources

Clearly, space communications will be dependent upon the availability of electrical energy and power sources. This factor may in many cases be a limiting one in the useful lifetime of a space vehicle, as in the case of the initial earth satellites. For a good survey of the characteristics and capabilities of present and possible future electrical energy sources see Huth.[8]

Radio-Frequency Power Sources

We consider here particularly the improvement of the capabilities of solid-state RF power sources, which are intrinsically more efficient than those which require filament power. At present, the power capabilities of solid-state devices are severely limited, especially at frequencies in the range of 100 mc or higher.

Also, applications can be envisaged[9] in which it is desired that the transmission consist of relatively short (say, millisecond) pulses of relatively high (kilowatt) peak powers. Present-day tubes capable of delivering kilowatts of peak power must draw substantial filament power for times much longer than a millisecond, thus resulting, in such cases, in very low efficiency.

Data Storage and Data Encoding

It has been stated above that the requirement for data storage may arise, for example, from line-of-sight considerations, or for purposes of retransmission at a slower rate, reducing required channel capacity.

Improved data-encoding methods will yield channels which are closer to the maximum transmission rate.

Low Noise Receivers

This refers to such devices as masers.

[8] J. H. Huth, "A Discussion of Energy Sources for Space Communications," The RAND Corp., Paper No. P-1318; March, 1958.

[9] P. Swerling and C. M. Crain, "A Possible Transponding System for an Artificial Asteroid," The RAND Corp., RAND Memo. No. RM-2172; May, 1958.

Directive Vehicle Antennas

It has been seen that the use of a vehicle antenna having a gain of, say, 100 or 1000 can have a very large payoff in terms of required power, or, alternatively, in terms of possible channel capacity, necessary ground antenna size, etc. There would be imposed the requirement that the vehicle antenna beam be stabilized and capable of tracking the other end of the communication link. Methods for doing this would appear to be a most promising avenue for development. Nonmechanical (electronic) beam steering may be particularly important in this connection.

Very Large Surface Antennas

Radio astronomy, the air defense surveillance net, and current space-flight tracking activities are building up a backlog of experience in the use of very large steerable ground antennas, which are necessary to many space communication missions.

In this connection, one might mention two limitations on the useful size of ground antennas. First, if the communication is supposed to penetrate the earth's atmosphere, the atmosphere will cause phase distortions of the wave front.[10] These distortions have the effect that, at any given frequency, a receiving antenna, beyond a certain size, will not realize its full theoretical receiving area. It has been estimated, for example, that the maximum useful size of receiving antennas due to this factor is about 500-foot diameter for a 10,000-mc frequency. A second limitation arises from the fact that very large antennas cost a large amount of money.

Circuit Components

This category is meant to include such items as improved miniaturization and packaging of components; greatly improved reliability for unattended operation up to perhaps several years; and investigation of, and protection from, damage caused by meteoric impact or radiation in space.[11]

Research on Cosmic, Solar, and other External Noise Sources

This has been discussed above. Continued research in this field is necessary with a view to compiling the most complete maps possible of noise intensity as a function of frequency and direction, and for different states of solar activity. One could also investigate what, if any, difference in noise intensity there would be outside the atmosphere.

Research on Atmospheric Physics of the Solar System

The fact that the earth's gaseous atmosphere and ionosphere crucially affect present-day communication is obvious. In like manner, the atmospheres and ionospheres of other bodies in the solar system will affect communication to or from the surface of these bodies.

In this connection one might mention the region of space of most immediate interest, that is, the earth-moon system. Although the moon has a negligible gaseous atmosphere, electron densities in the neighborhood of the moon may reach the respectable values of 10^6 or 10^7 per cubic centimeter,[12] which would certainly affect communication in this region.[13] Even in the space between earth and moon, electron densities may be of the order of 10^3 per cc. This affects, among other things, the velocity of light in earth-moon space. More precise measurement of electron densities in these regions constitutes an immediate possibility for useful research of this type.

Search and Acquisition Techniques

If continuous tracking of the space vehicle is impossible for any reason (such as intermittent radiation from the vehicle), it is necessary to search for and acquire the vehicle if communication is to resume. In many cases, simultaneous search in direction and frequency may be necessary.

Very large ground antennas have correspondingly small beamwidths; consequently auxiliary searching antennas with wider beams may be necessary in some cases. Also, precise trajectory predictions will be very important in reducing the area of sky to be searched.

In the frequency domain, for communication frequencies of interest (say a few hundred to a few thousand megacycles), and for typical radial velocities (or the order of tens of kilometers per second), Doppler shifts will be of the order of kilocycles or tens of kilocycles, while it may be desired to use a channel bandwidth of, say, one hundred, ten, or even one cycle per second. In other words, Doppler shifts may be many times as large as the desired channel bandwidth.

Precise trajectory predictions will enable one to reduce greatly the unknown component of those Doppler shifts. Even if the unknown component remains larger than the desired channel bandwidth, methods are available for searching for and locking onto a narrow band signal when the uncertainty in location exceeds the bandwidth.

Additional Factors

The above list is not, of course, exhaustive. As an illustration of some of the other factors which must be considered by the space communication system designer, one might mention the choice of frequency.

To illustrate, consider the case of communication between the earth's surface and a point beyond the earth's atmosphere. For frequencies above about 30,000 mc,

[10] C. M. Crain, "Survey of airborne microwave refractometer measurements," Proc. IRE, vol. 43, pp. 1405–1411; October, 1955.

[11] J. J. Harwood, H. H. Hausner, J. G. Morse, and W. G. Rauch, "Effects of Radiation on Materials," Reinhold Publishing Co., New York, N. Y.; 1958.

[12] Some estimates are as high as 10^{10} per cubic centimeter.

[13] S. Chapman, "Notes on the solar corona and the terrestrial ionosphere," *Smithsonian Contributions to Astrophysics*, vol. 2, pp. 1–14; 1957.

there will be very large atmospheric absorption losses, while below about 10 mc the radiation will be reflected by the ionosphere. Other atmospheric effects must also be taken into account. For example, below about 1000 mc the plane of polarization of the radiation will be rotated in the ionosphere by some unpredictable amount, causing polarization losses, unless one uses, say, circularly polarized antennas. (Analogous considerations apply to communication to and from the surfaces of extraterrestrial bodies.)

On the other hand, the noise level is also affected by the choice of frequency, in the case of both cosmic and solar noise and internal receiver noise.

Also, in choosing frequency, an eye must be kept on the fact that, at any given time, the availability of components having desired characteristics is definitely dependent upon operating frequency.

It is likewise of interest to mention relativistic effects on the various communication equations. If $\beta = v/c$, where v represents vehicle radial velocity (positive for approaching, negative for receding) and c represents the velocity of light, then[14]

$$\frac{(S_r)_{\text{moving}}}{(S_r)_{\beta=0}} = \frac{(1+\beta)^2}{1-\beta} \tag{5}$$

$$\frac{(W)_{\text{moving}}}{(W)_{\beta=0}} = \sqrt{\frac{1+\beta}{1-\beta}}\,. \tag{6}$$

[14] E. Rechtin, "Space communications—feasibility," Lectures on space technology, Univ. of Calif., Berkeley; 1958.

Also, the Doppler shift in frequency f is given by

$$\frac{f_{\text{moving}}}{f_{\beta=0}} = \sqrt{\frac{1+\beta}{1-\beta}}\,. \tag{7}$$

At least for the first phases of space flight (say, exploration of the solar system), relativistic effects will be negligible. The one possible exception is the measurement of radial velocity by Doppler shift. For example, if $f_{\beta=0}$ were 1000 mc, and v were 30 km/sec, one would have to use the relativistic expression in order to convert f to β, if an accuracy of about one in 10^{-4} or better were desired.

One should not leave this subject without taking note of some speculations on using the space environment to facilitate space communications. Among such speculations are the following:

1) Taking advantage of the natural high vacuum that exists in space.
2) Taking advantage of the large temperature differences and near-absolute-zero temperatures available.
3) Taking advantage of the absence of gravity in building and steering large structures, such as antennas.

It should be noted that the use of space vehicles as aids to earth communications is growing beyond the speculative stage.

In closing, let me add that, in all probability, the most important novel interactions between communications and space flight are unpredictable at present.

Self-Contained Guidance Systems*

CHARLES S. DRAPER†

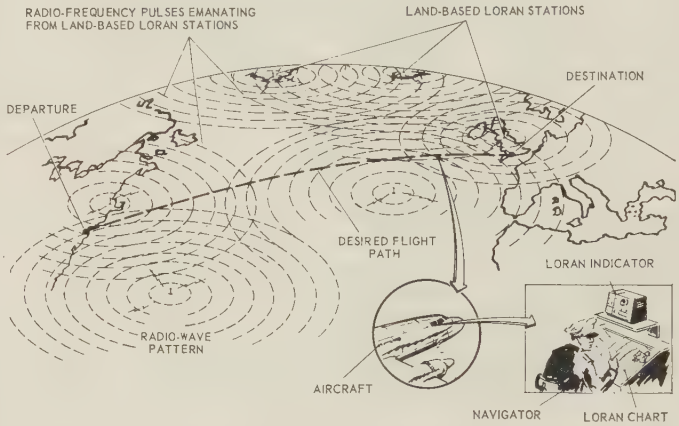
Summary—**Inertial guidance is based on the use of reference coordinates established by applications of Newton's Laws of Motion to self-contained systems. Gyro units carried by servo-powered gimbals give accurate information on preset angular positions that may be used to supply the function of the celestial sphere in conventional navigation. Changes in position are indicated by integration of acceleration components along axes fixed to the gyro-stabilized member. When vehicles moving over or near the earth's surface are involved, the use of Schuler tuning to give vertical indications unaffected by linear acceleration makes accurate long-range inertial guidance possible. Because in all cases only the laws of gravity and classical mechanics are involved, inertial guidance systems are generally free from interference, short of actual physical damage. Systems based on gyro units and accelerometers are made possible only by modern developments in mechanical design, materials, electronics, and servomechanism techniques. Illustrative examples are given and discussed to bring out the nature of the problems involved. Beyond question, the future holds many applications of inertial principles to a wide range of guidance problems.**

Introduction

GUIDANCE is the process of causing an object to follow some desired path with respect to selected reference points. Various methods for guidance are possible, the essential differences being in the coordinate systems that are used and the means employed to establish positions in these systems. *Direct-radiation-contact guidance* is carried out by means of electromagnetic- or sonic-wave connections between the guided entity and the reference points. *Auxiliary-reference guidance* uses one or more coordinate systems in addition to the space directly determined by the selected reference points and the guided entity. Fig. 1(a) illustrates the simplest possible guidance situation, with direct visual line-of-sight contact between a man and a cave as his destination. Here the single set of coordinates fixed to the man's head is the only space involved. Fig. 1(b) illustrates direct-radiation-contact guidance when optical radiation is replaced by radio wavelengths and reference points fixed to the earth are established by a number of stations. Arrangements of this kind make it possible to accomplish guidance when the reference points and the guided entity are all beyond line-of-sight contacts, or when poor visibility eliminates visual observations.

The essential result of using radiation contacts is that the *present position* of the *guided entity* is known. Guidance itself is the process of changing the direction and magnitude of the motion of the guided entity so that the present position approaches the desired position. Fig. 2 illustrates the guidance corrections for a typical instant when the present position is not on the desired track, so

(a)



(b)

Fig. 1—Guidance in terrestrial coordinates when a clear line of sight exists or a direct radiation connection is available. (a) Destination directly visible to man. (b) Earth coordinates and a typical reference coordinate system.

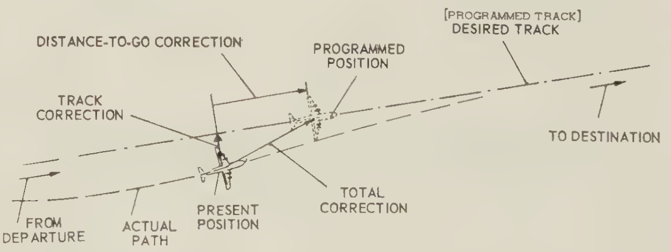


Fig. 2—Guidance corrections.

that a *track correction* and a *distance-to-go correction* are required to bring the guided entity into coincidence with the *programmed position*. A perfect guidance system would keep both corrections substantially at zero.

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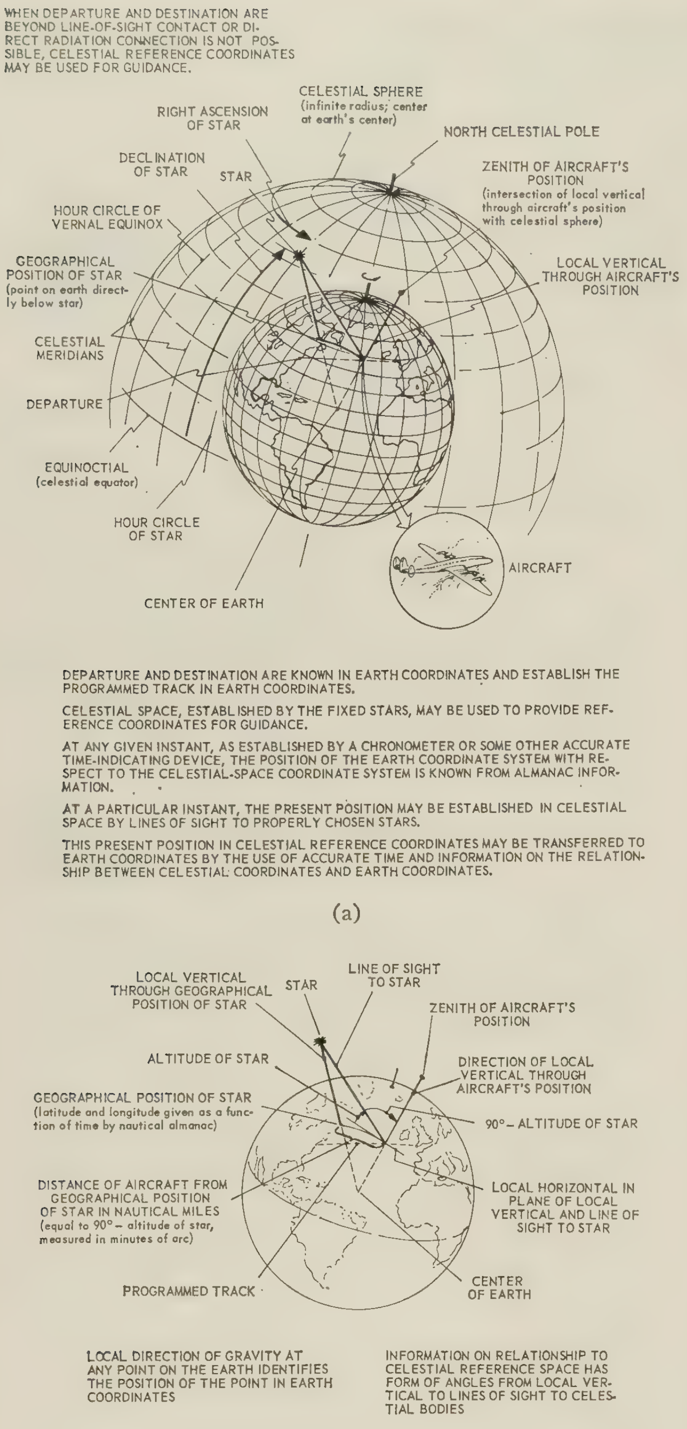


Fig. 3—Guidance by use of celestial reference coordinates when destination is beyond line-of-sight contact or direct radiation connection. (a) Present position in earth coordinates by transfer from celestial reference coordinates. (b) Angles between celestial-body lines of sight and local direction of gravity gives present position in celestial reference coordinates. (See Dutton [1].)

Direct-radiation-contact guidance is very often not possible for a number of reasons. The necessary equipment may not exist or may not be working because of natural or man-made interference. Lines of sight may not be available because of weather conditions, obstructions, lack of landmarks, or earth's curvature. When unfavorable radiation-contact conditions exist for any of these reasons, the celestial sphere may be used to provide auxiliary coordinates for guidance purposes. This, of course, is possible only when line-of-sight contacts with suitable stars are available. This use of stars is very old and is the basis for the universally practiced method of celestial navigation [1].

Fig. 3(a) illustrates the determination of present position by the use of star lines of sight and celestial reference coordinates. Fig. 3(b) shows that the essential data determined by observations are the angles between the local direction of gravity at the present position and a line of sight to a properly selected star. Figs. 4 and 5 illustrate the way in which a sextant or an octant is used by a human observer to measure the line-of-sight data needed to fix present position with respect to the celestial sphere.

The instantaneous position of the latitude and longitude coordinates of the earth with respect to the celestial sphere is fixed by the instant of time at which observations are made. By use of almanac data on stars and a knowledge of time from chronometer readings, it is a routine matter to transfer present position from celestial coordinates to earth coordinates.

Inertial Reference Coordinates for Guidance

Celestial space does not provide reference coordinates that are completely satisfactory for guidance purposes because they are available only under the special circumstances that allow reliable observations of stars. Overcasts, noctilucent clouds, daylight, too-close proximity to solar-system bodies, aerodynamic boundary layers on the guided vehicle, and imperfect windows combine to make celestial-body observations inaccurate or impossible. Special equipment may partially overcome these troubles by the addition of some complexity, but it is not likely that a completely satisfactory solution will be reached in equipment of reasonable size and cost.

The line-of-sight observation difficulties that are associated with the use of celestial space to provide auxiliary reference coordinates for guidance purposes may be eliminated by the use of instrumentally established inertial reference coordinates. This approach is feasible because Newton's Law of Dynamics [2] shows that the acceleration of a mass particle in response to an applied force occurs with respect to inertial space, which is effectively identical with celestial space. Gyroscopic rotors make it possible to establish an artificial inertial reference space within vehicle-carried guidance equipment. This inertial reference space may be given any desired orientation in celestial coordinates and has the very great advantage of giving continuous operation for guidance purposes, whether or not lines of sight to celestial bodies are available. Fig. 6 illustrates the basic features of a configuration in which an *inertial reference package* holds an initially established orientation with respect to celestial space. This orientation is shown with

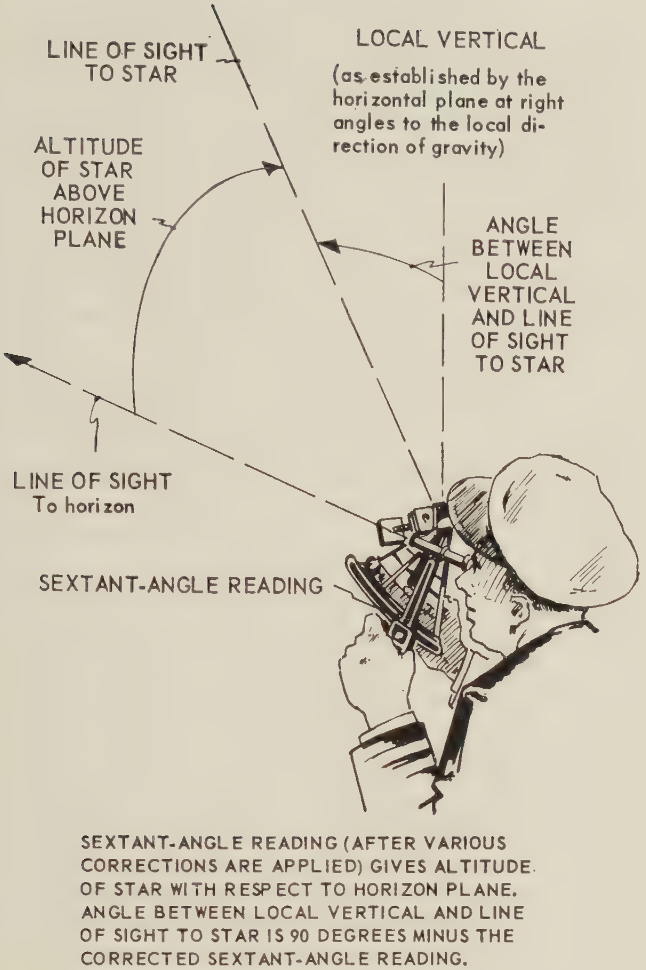


Fig. 4—Navigator's measurement of angles between celestial-body lines of sight and local direction of gravity.

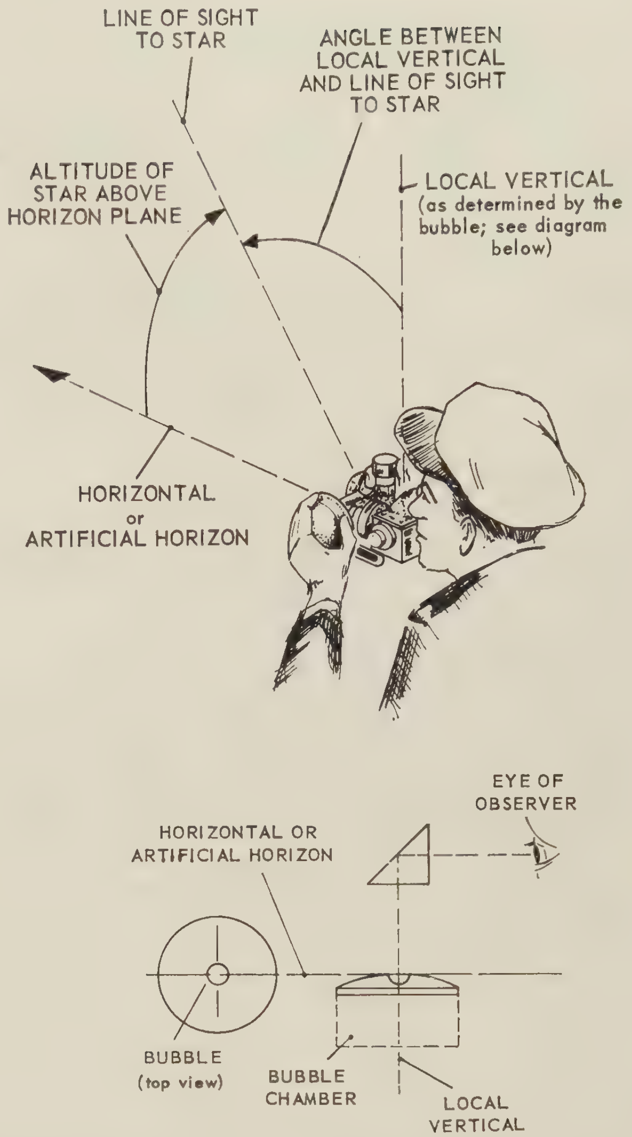


Fig. 5—Use of bubble octant to establish angle between celestial-body line of sight and local direction of gravity.

the supporting axis of the package set parallel to the earth's axis of rotation.

With the arrangement of Fig. 6, two auxiliary spaces are effectively applied in solving the guidance problem: inertial reference coordinates and earth reference coordinates. The inertial reference coordinates are fixed to the mechanical component that carries the inertial-reference package. These coordinates are given an initial orientation with respect to celestial coordinates and are held in this orientation by components inside the package. The inertial space established by this instrumental means provides the auxiliary reference coordinates for guidance purposes. The conversion from these inertial reference coordinates to earth reference coordinates is especially simple when the support axis for the inertial-reference package is set parallel to the earth's axis. This is true because with this arrangement it is only necessary to supply an accurate sidereal time signal to the drive between the package and the gimbal. This drive then causes the gimbal to be so rotated that it always remains parallel to some meridian of longitude on the earth. Thus the gimbal may be used as the *earth reference member* from which positions may be found in terms of angles between the local direction of gravity and a direction fixed with the proper orientation to the reference member. Various configurations for realizing indications of present position by inertial-guidance equipment are possible, but the basic principles involved in each case are those just described.

Satisfactory inertial-guidance equipment for vehicles to be guided with respect to references that are located in earth coordinates depends on the practical realization of:

1) Means for maintaining the orientation with respect to inertial space of the inertial-reference package carried by a moving vehicle within tolerance limits and over time periods that are compatible with the requirements of the guidance problem to be solved.
2) Means for accurately indicating the local direction of gravity within equipment subjected to the erratic rotations, linear accelerations, and gravitational components that accompany the operation of a moving vehicle.

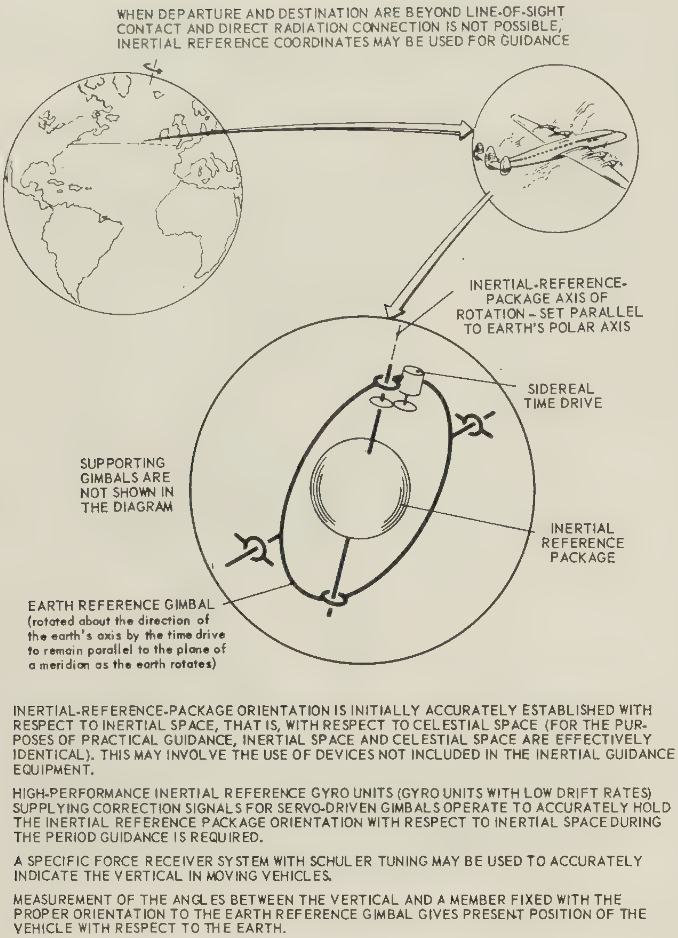


Fig. 6—Guidance by substitution of inertial reference coordinates for celestial coordinates.

The first of these requirements may be met by a servo-driven gimbal system and gyro units with proper performance as far as the ability to hold orientation with respect to inertial space is concerned. Units with this low-drift-rate characteristic are of primary importance for inertial-guidance equipment. Reduced to the simplest terms, inertial guidance is a refined form of dead-reckoning navigation in which good accuracy of present-position indications depends on inertial-reference-package coordinates that within certain tolerance limits maintain their orientation with respect to inertial space.

The second special inertial-system requirement applies particularly to equipment for use in winged or floating vehicles that ordinarily operate with gravitational-field effects balanced by externally applied forces. The instrumental problem to be solved is that of accurately determining the direction of gravity in the presence of linear accelerations that vary erratically in direction and magnitude. This is difficult to accomplish in practice because of the physical fact described by Einstein's [3] principle of equivalence, which states that, in general, it is impossible to distinguish body force components due to linear acceleration from body force components due to gravitational-field effects. In the face of this situation, satisfactory indications of the local vertical, which by definition is identical with the direction of gravity as this direction would be indicated by an accurate plumb bob suspended from a point fixed to the earth, may be achieved by designing the proper dynamic characteristics into the vertical indicating subsystem. The nature of these characteristics and their theoretical background have been described by Schuler and others in a series of articles [4]–[8]. Schuler's great contributions to the field of inertial guidance are universally honored by describing the essential dynamic characteristic of vertical-indicating subsystems as *Schuler tuning*.

Inertial-guidance equipment that must operate in environments that prevent the direction of gravity from being used as an input obviously cannot be designed to use this direction for determining position. For example, the orbiting flight of a satellite causes gravity to be exactly counterbalanced by centrifugal force, so that any specific force receiver such as a pendulum would have zero input and could not be used to establish the direction of gravity. A similar condition would exist in the unpropelled portion of a ballistic-missile flight or in a freely falling airplane. Interplanetary or moon-travel vehicles would also be substantially free from gravity for part of their trajectories so that the "direction of gravity with respect to the earth" would have no practical significance. In certain cases, variations in the gravitational gradient may be used for establishing a direction, although this effect is very much weaker than that of gravity acting on an unbalanced mass supported on a stationary base.

Integration of specific-force components with respect to known inertial reference coordinates may be used for inertial-guidance purposes when gravitational force direction is not available. Systems based on specific-force-component inputs also have certain advantages for guidance from gravity-direction changes. Wrigley, Woodbury, and Hovorka [9] give a discussion of the various configurations that may be used for practical equipment and indicate the performance characteristics to be expected.

The basic problems of low-drift-rate gyro units and other special components for inertial guidance systems are described in various sources [10]–[13].

General Features of an Inertial Guidance System

Inertial-system operation depends on the availability of four mechanism components:

1) An *inertial reference package*, which provides signals depending on the deviation of the package from an orientation that is nonrotating with respect to inertial space.

2) A *specific force receiving package*, which provides signals depending on the resultant of gravitational forces and the inertia-reaction forces due to the linear acceleration with respect to inertial space that acts on the package.

3) A *time signal generator*, which gives an output accurately representing sidereal time.

4) An *indicating system*, which shows the change in direction of the local vertical with respect to a direction related to the point of departure by the inertial-reference package and sidereal time, and generates guidance information by comparing the angle between these two directions with a guidance program.

A complete description of all the possible system types would be out of place in the present discussion of principles. To illustrate the kinds of problems that are associated with the practical realization of inertial guidance, a system employing the earth's gravity field direction for use between terrestrial points will be used for the purpose of describing principles.

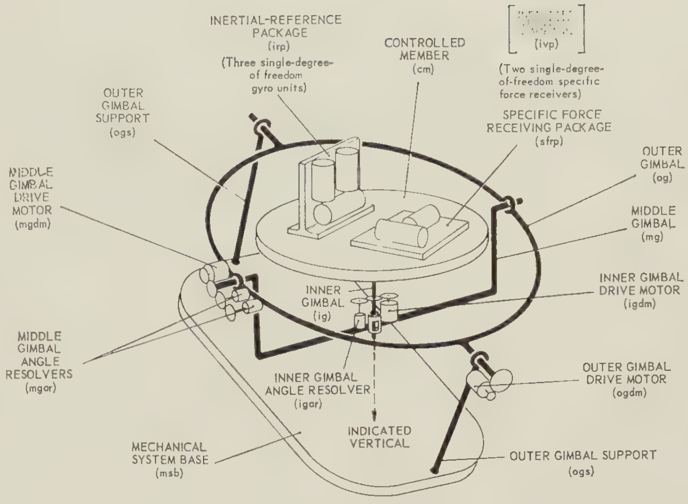
Mechanical Features of an Illustrative Inertial Guidance System

Fig. 7 is a line-schematic diagram showing the essential elements of the mechanical subsystem for an illustrative inertial guidance system in which the inertial-reference package and the specific force receiving package are both rigidly connected to a controlled member. This controlled member is mounted on a base by means of a three-gimbal system that allows the controlled member complete angular freedom with respect to the base. The system includes three single-axis servomotor drives: one between the outer gimbal support and the outer gimbal; one between the outer gimbal and the middle gimbal; and one between the middle gimbal and the inner gimbal, which carries the controlled member. A system of this kind is discussed by Draper and Woodbury [14].

The *specific force receiving package* of Fig. 7 consists of two single-direction-sensitivity specific force receiver units with their input axes at right angles to each other in a plane perpendicular to the axis of the inner gimbal. This package has the function of receiving the specific force inputs that form the basis for indicating the direction of gravity, and for this reason is also called the *indicated vertical package*.

Fig. 8 is a functional diagram for an inertial guidance system based on the mechanical arrangement of Fig. 7. Each of the three gyro units of the inertial-reference package, by interactions among its internal components, generates a signal proportional to the inertial-space rotation of its case about the input axis with respect to a *reference orientation* of the case. This reference orientation is nonrotating with respect to inertial space unless changed by an input command signal to the gyro unit.

The angular-stabilization loop of Fig. 8 operates to keep the change in orientation of the controlled member effectively identical with the change in the reference orientation of the inertial-reference package. This means that the components that make up the stabilization loop act as a three-axis *inertial-space angular integrating system*, which may be also called a *space integrator* when a misunderstanding is not likely.



Note: 1) The base motion isolation gimbal system is made up of the outer gimbal supports, the outer gimbal, the middle gimbal, the inner gimbal, the associated drive motors, and the associated resolvers.
2) The electrical power supplies, electronic units, computers, connections, racks, and other components necessary to complete an inertial guidance system are not represented in this figure.
3) This illustration is based on Fig. 4 of Wrigley, Woodbury, and Hovorka [9] and Fig. 6 of Draper and Woodbury [14].

Fig. 7—Line-schematic diagram showing the essential mechanical elements of an inertial guidance system based on rotation of the inertial-reference package with the indicated vertical.

The two units of the specific force receiving package shown in Fig. 7 are rigidly attached to the controlled member so that they generate signals proportional to the specific forces acting along two directions at right angles to the inner-gimbal axis. These two signals form a composite specific-force signal representing the resultant of linear-acceleration forces in a plane normal to the axis of the inner gimbal and the projection of gravitational force on this plane. This specific-force signal is the input for the *indicated vertical drive signal generator*, which supplies the input signal for the space integrator. The response of the space integrator to this signal is to rotate a direction fixed to the controlled member into substantial coincidence with the *local direction of gravity*, that is, the direction in which an accurate plumb bob would hang on a stationary base at the instantaneous location of the guidance system.

The gravitational-force components act to correct the indicated vertical, but the horizontal-plane linear-force components, which in modern aircraft are of considerable magnitude in comparison with gravity and continue for relatively long time intervals, act to cause intolerably great errors of the indicated vertical unless the equipment involved is properly designed. *The basic problem that must be solved by a satisfactory vertical indicating system is that of compensating for the interfering effects of linear-acceleration disturbances, so that the indicated vertical accurately follows the local direction of gravity when the system is carried by a moving vehicle as well as when it is stationary on the earth.* The solution is provided by Schuler tuning.

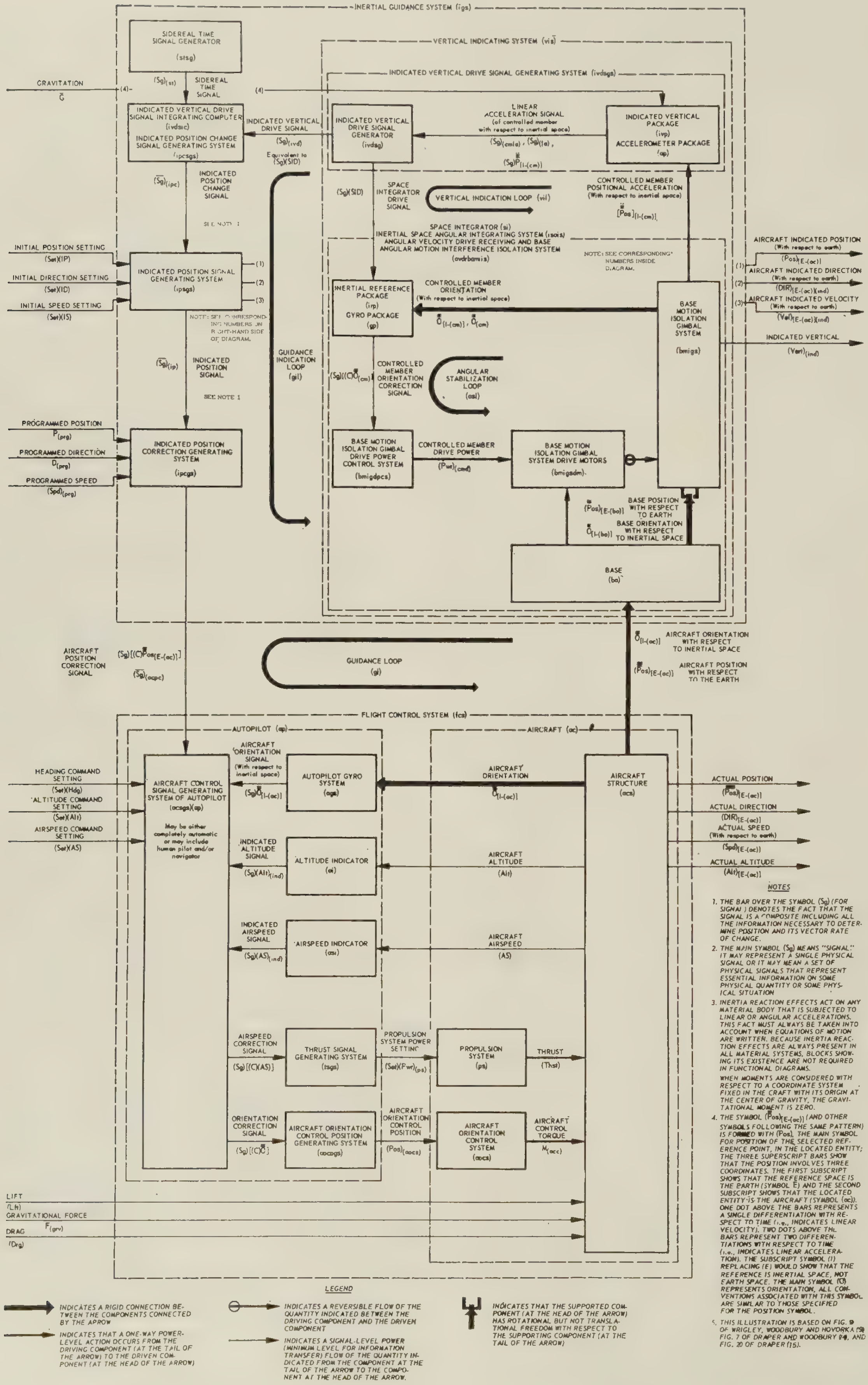


Fig. 8—Functional diagram showing essential subsystems and components of an illustrative inertial guidance system based on continuous alignment of the inertial package with the indicated vertical.

As shown by Fig. 8, the *vertical indicating system*, which is formed by the *indicated vertical drive signal generating system* and the *space integrator*, has the position and orientation of the aircraft as its inputs and produces the *indicated vertical* and the *indicated vertical drive signal* as its outputs. A necessity for realizing inertial guidance in practice is the provision of an integrating computer as part of the equipment. This computer should have the same integration performance as the space integrator containing the inertial-reference package. By supplying the integrating computer with the indicated vertical drive signal as its input, an output representing the resultant angular change in the direction of the indicated vertical during any elapsed time period will be generated. The signal representing this angular displacement will be a measure of the distance moved by a craft carrying the equipment over a nonrotating earth. With the equipment stationary on the actual earth, which is rotating at the rate of one revolution in twenty-four sidereal hours, the integrating computer will give an output corresponding to an indication of sidereal time. Because the angular velocity of the earth is accurately known, it is possible to subtract the effect of the earth's rotation from the integral of the indicated vertical drive signal on the basis of an input to the integrating computer from a sidereal time signal generator.

The indicated vertical drive signal integrating computer of Fig. 8 produces a signal whose change during any given time interval represents the change in indicated position with respect to the earth during the same interval. This indicated-position-change signal is the primary input for the *indicated position signal generating system*, which also accepts settings of *initial position*, *initial direction*, and *initial speed*. This system combines the information on indicated-position change with information on starting conditions to produce output signals that continuously represent the position, the direction, and the speed of the aircraft carrying the guidance system

The *indicated-position signal* is one of the inputs to the *indicated position correction generating system*, which also receives programmed position, programmed direction, and programmed speed from a source outside of the system represented by the functional diagram of Fig. 8. This system compares instantaneous states of the programmed quantities with the actual states of the same quantities and produces a composite signal representing the corrections to the actual quantities that are required to bring them into agreement with their programmed states.

Fig. 8 shows that the output of the *inertial guidance system*, which functionally combines the stabilization loop and the vertical indication loop to form the *guidance indicating loop*, is the *aircraft position correction signal*. This signal contains the information necessary to change the direction and speed of the aircraft so as to cause it to approach the programmed state of these quantities. The aircraft position correction signal is the primary input for the autopilot that produces the inputs necessary to operate the orientation control system of the aircraft and to adjust the power setting of the propulsion system. The *flight control system*, whose basic function is to determine the path and speed of the vehicle carrying the inertial guidance system, is made up of the autopilot and the aircraft itself.

Geonavigational Factors—Shape of the Earth [16]–[18]

According to Newton's law of gravitation, a massive body such as the earth can be considered as having associated with it a gravitational field. The spatial direction of this field at any given point on the earth might serve as a unique identification of the position of that point. However, it is impossible to distinguish directly between gravitational forces and inertial forces, such as the centrifugal force due to the earth's daily rotation. This is expressed in a basic law of physics, namely the principle of equivalence in the general theory of relativity, that gravitational mass and inertial mass are equivalent [19].[1] The vector resultant of the earth's gravitational field and the centrifugal force per unit mass due to the earth's daily rotation is defined as the earth's gravity field (see Fig. 9). The spatial

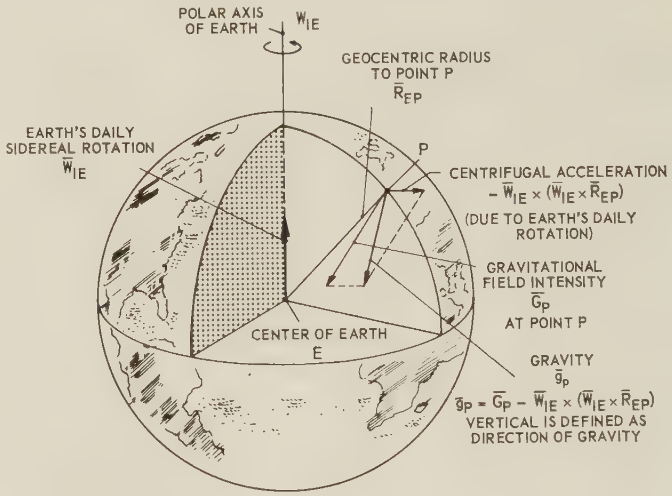


Fig. 9—Relationship of factors to produce the vertical.

direction of this gravity field is nearly radial and, neglecting anomalies, is unique at any given point on the earth. The secular (time) variation in the direction of the gravity field at a given point, caused mainly by tidal effects, is less than 0.05 μr [16]. Thus, for navigational purposes, the direction of the gravity field is a reliable, unique characteristic of any given point on the earth.

[1] Here, *gravitational mass* is the mass concept inherent in the inverse-square law of mass attraction, while *inertial mass* is the mass concept inherent in Newton's second law, that the net force is given by the product of mass and acceleration.

Furthermore, it is essentially impossible to interfere with gravity effects.

The earth itself is also subject to the equivalence of gravitational and inertial effects. Because of daily rotation, the associated centrifugal force field causes the earth to bulge at the equator, producing an ellipsoidal shape. The figure that a fluid body with the mass distribution and daily rotation of the earth would have is defined as *the geoid.* The surface of the geoid is represented by mean sea level, and variations in the elevation of the geoid relative to the closest reference ellipsoid are approximately one per cent of the topographic variations in elevation. The reference ellipsoid, having an ellipticity[2] of 1/297, is seen to depart only slightly from a sphere. *The direction of the force of gravity, which is the gradient of the gravity potential at the surface of the geoid, is defined as the vertical;* it is illustrated in Fig. 10. It is indicated most simply by a plumb bob whose base is stationary with respect to the earth.

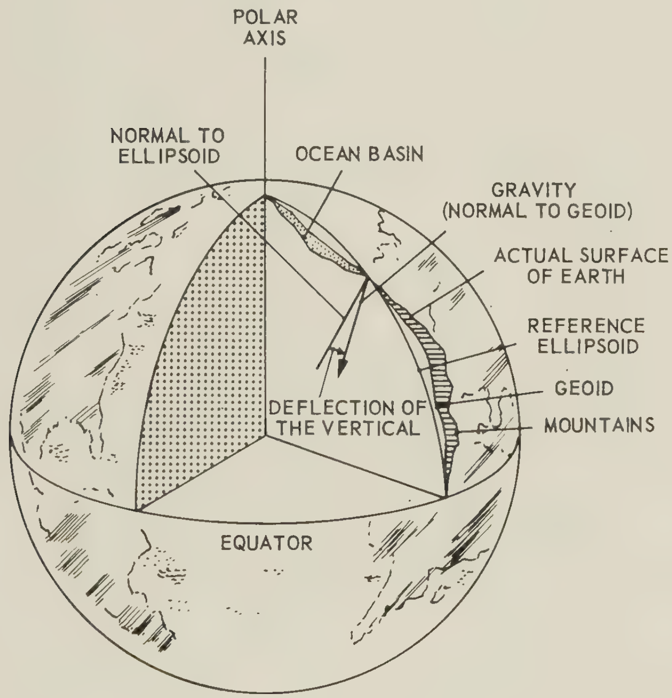


Fig. 10—Relationship of the geoid and a reference ellipsoid.

The geoid is not analytically smooth, because of local variations in the densities of the materials that make up the earth's crust. This is further accentuated by the topographic variations in the earth's surface. Such deviations are known as *gravity anomalies.* Because the geoid does not have an analytically smooth surface, the vertical is not in general parallel to the normal to the reference ellipsoid at the same position. This angular deviation, called *the deflection of the vertical* or *station error,* is generally less than 0.30 mr, and over a continental land mass rarely exceeds 0.10 mr. Fig. 10 shows the relationship of the geoid to a reference ellipsoid.

[2] Ellipticity is the ratio of the difference between the equatorial radius and the polar semidiameter to the equatorial radius.

Schuler Tuning

The Schuler tuning that is required for accurate indication of the vertical on any moving base can be achieved in practice by making the force-sensitive properties of the vertical indicating system separate from the torque-producing properties, in two distinct subsystems, and inserting a dynamic control function between them. When the dynamic control function is an integrator, the vertical indicating system performance is of second order and undamped, exhibiting forced dynamic acceleration error. By modifying the dynamic control function in one of several ways, however, the necessary damping can be obtained.

In the simplest case, the dynamic control function is a direct coupling and the performance is describable by a first-order differential equation. This is the artificial horizon, which acts like a heavily over-damped pendulum, and cannot be Schuler tuned. Its performance, which for typical data is shown as Case *a* in Fig. 11, is not suitable for inertial guidance.

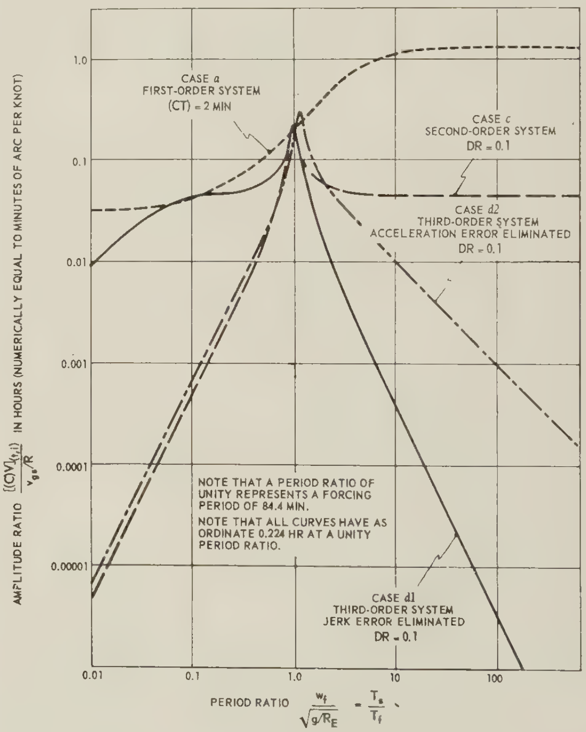


Fig. 11—Steady-state amplitude ratio–period ratio response for vertical indication loops.

If a bypass is added around the coupling integrator, the loop is damped, and will settle to a steady state. However, the addition of the bypass, while producing damping, also results in a forced dynamic error. This method is shown in Case *c* of Fig. 11.

The preceding system is a "noisy" system, which can be markedly improved by the addition of a lag network. The immediate result of this more elaborate coupling

scheme is a choice between two Schuler-tuning methods:

1) In one case, the acceleration error is eliminated.
2) In the other case, the jerk error is eliminated.

The performance obtained from these two methods is shown by Cases *d1* and *d2*, respectively, of Fig. 11. It can be seen that Case *d1* represents the best performance. This case corresponds to the "differentiated-tachometer-feedback" type of damping found to be so effective with servomechanisms.

Space-Integrator Performance

Space integration is the process of receiving signal inputs and producing outputs in the form of corresponding angular-velocity components with respect to inertial space. Effective performance of this function is possible only if the system involved is insensitive to electrical and mechanical disturbing effects. Electrical interference is reduced to tolerable levels by the usual techniques of design, shielding, and installation. Mechanical interference may be due to external torque components acting directly on the controlled member or to base oscillations that tend to move this member by inertial coupling and support-bearing torques that act against motion between the base and the controlled member. Only these bearing torques are present under static conditions, while both bearing torques and inertial coupling occur when the base is rotating. The essential features of the space-integrator-control-system problem are illustrated by the pictorial and line-schematic diagrams of Fig. 12. This figure actually represents a test table for single-degree-of-freedom gyro units with a servo-drive supplied with power from an electronic amplifier on the basis of signals from the gyro unit under test.

Fig. 13 is a functional diagram for the gyro-test-table system. The base supports the controlled member shaft, which also carries the rotor of the drive motor. The gyro unit is fixed to the controlled member, with its input axis parallel to the axis of rotation of the controlled-member shaft. The gyro unit is shown as receiving the orientation of the controlled member, the input command signal, and the modifying inputs that correspond to the excitations and environmental-control quantities necessary to keep the unit operating properly. The output signal from the gyro unit, which represents the angular correction to the orientation of the controlled member required to bring the actual orientation of the output axis into coincidence with the reference orientation of the output axis, is the input for the *controlled member drive motor power control system.* This system consists of a number of components including: the ac preamplifier, the demodulator, the signal modifier, the dc amplifier, the modulator, the ac post-modulator amplifier, and the motor drive power amplifier. The output current from this latter amplifier is the input for the controlled member drive motor and interacts with the drive-motor excitation to apply torque to the controlled member.

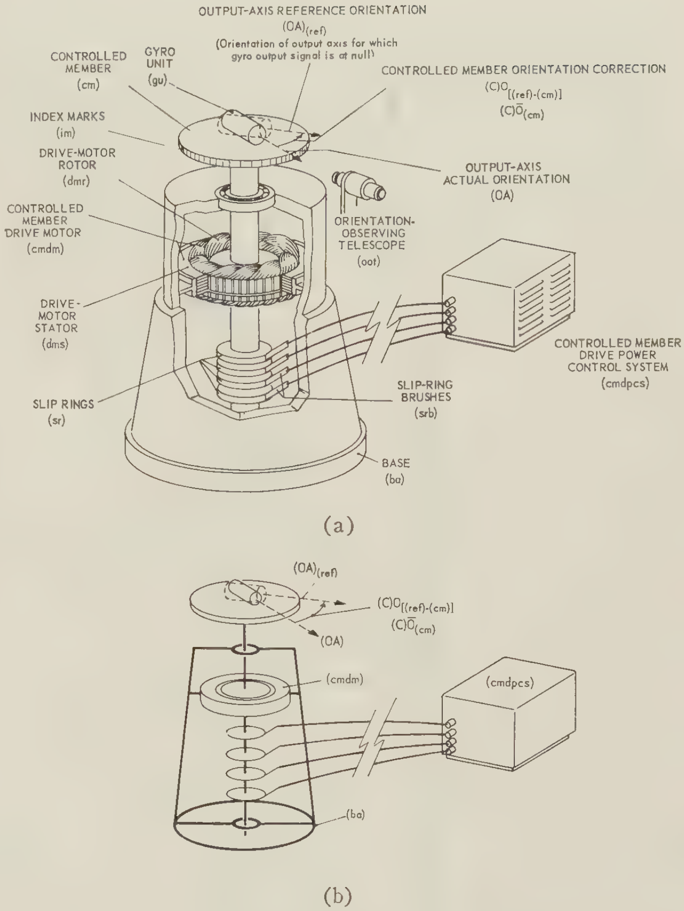


Fig. 12—Pictorial diagram showing the elements of an illustrative single-axis controlled member inertial space geometrical stabilization system. (a) Elementary pictorial diagram of single-axis servo-table gyro test equipment. (b) Line schematic of single-axis servo-table gyro test equipment.

In practice, it is possible to achieve interference-torque input–angular-motion output sensitivities less than two hundredths of a milliradian per foot pound.

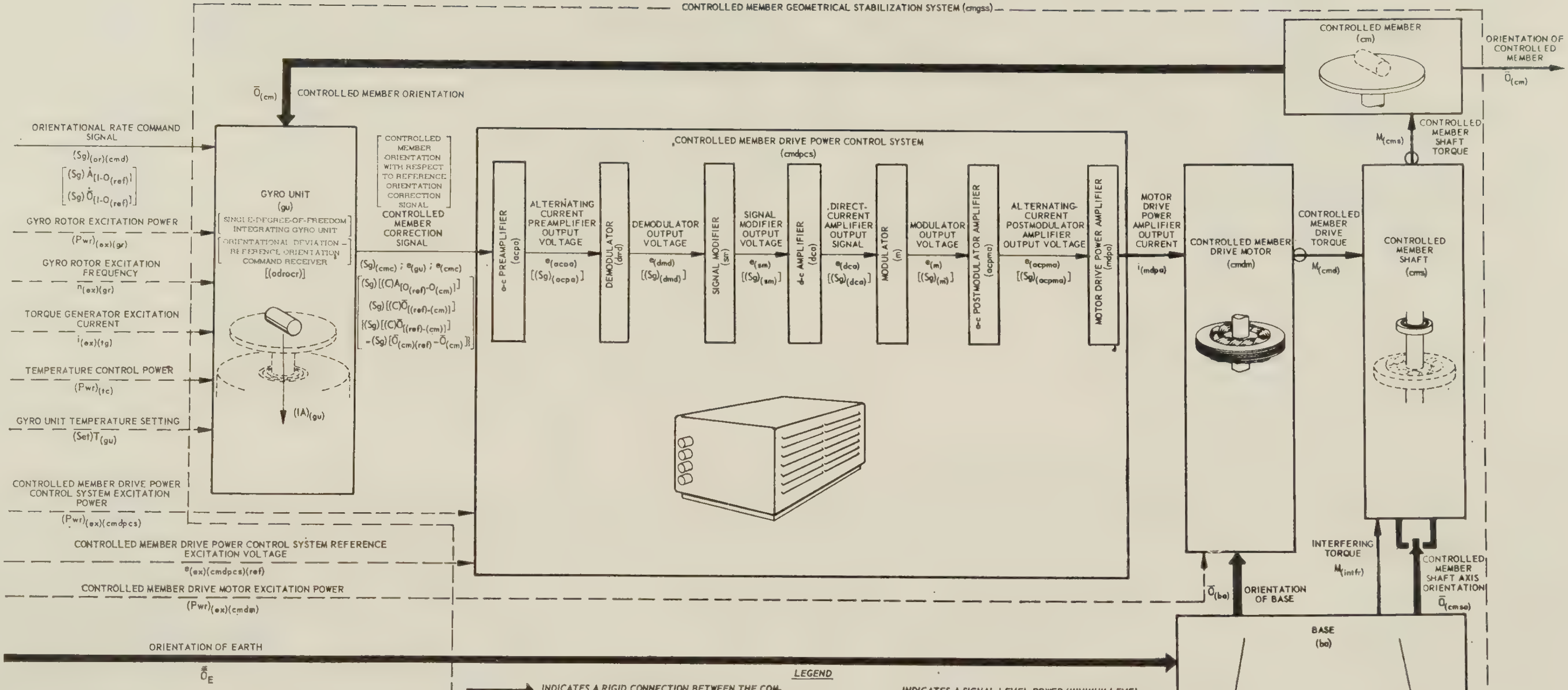
Fig. 14 shows typical amplitude-ratio response curves for practical space-integrator systems in terms of the equivalent undamped natural frequency of the controlled member geometrical stabilization system. It should be noted that:

1) The command-angle-input response holds near unity, its ideal value, to a higher frequency than the the frequency at which the interference response begins to drop off from its low-frequency level.

2) In any practical system, the reference level for this low-frequency response is several orders of magnitude down from unity. In order to simplify the plot, this reference is not shown at its actual position.

Conclusion

Inertial-guidance principles and problems have been outlined in the preceding sections of this paper. Detailed descriptions of specific items of equipment have been avoided in favor of generalizations that must be true for all inertial guidance systems. It must be considered that, at the present time, tests of a few proto-



NOTES

1. THE SYMBOL $(Sg)_{(\ )}$ REPRESENTS THE OUTPUT SIGNAL FROM THE COMPONENT IDENTIFIED BY THE SYMBOL PLACED WITHIN THE SUBSCRIPT PARENTHESES. FOR EXAMPLE, $(Sg)_{(pa)}$ REPRESENTS "PREAMPLIFIER OUTPUT SIGNAL."
2. INERTIA REACTION EFFECTS ACT ON ANY MATERIAL BODY THAT IS SUBJECTED TO LINEAR OR ANGULAR ACCELERATIONS. THIS FACT MUST ALWAYS BE TAKEN INTO ACCOUNT WHEN EQUATIONS OF MOTION ARE WRITTEN. BECAUSE INERTIA REACTION EFFECTS ARE ALWAYS PRESENT IN ALL MATERIAL SYSTEMS, BLOCKS SHOWING ITS EXISTENCE ARE NOT REQUIRED IN FUNCTIONAL DIAGRAMS.

   WHEN MOMENTS ARE CONSIDERED WITH RESPECT TO A COORDINATE SYSTEM FIXED IN THE CRAFT WITH ITS ORIGIN AT THE CENTER OF GRAVITY, THE GRAVITATIONAL MOMENT IS ZERO.
3. THIS ILLUSTRATION IS BASED ON THE FIGURES OF SECTIONS a AND b OF DERIVATION SUMMARY 8 OF DRAPER, WRIGLEY AND GROHE (10) TOGETHER WITH FIG. 21 OF THIS SAME REFERENCE.

LEGEND

- INDICATES A RIGID CONNECTION BETWEEN THE COMPONENTS CONNECTED BY THE ARROW
- INDICATES THAT A ONE-WAY POWER-LEVEL ACTION OCCURS FROM THE DRIVING COMPONENT (AT THE TAIL OF THE ARROW) TO THE DRIVEN COMPONENT (AT THE HEAD OF THE ARROW)
- INDICATES A REVERSIBLE FLOW OF THE QUANTITY INDICATED BETWEEN THE DRIVING COMPONENT AND THE DRIVEN COMPONENT
- INDICATES A SIGNAL-LEVEL POWER (MINIMUM LEVEL FOR INFORMATION TRANSFER) FLOW OF THE QUANTITY INDICATED FROM THE COMPONENT AT THE TAIL OF THE ARROW TO THE COMPONENT AT THE HEAD OF THE ARROW
- INDICATES THAT THE SUPPORTED COMPONENT (AT THE HEAD OF THE ARROW) HAS ROTATIONAL BUT NOT TRANSLATIONAL FREEDOM WITH RESPECT TO THE SUPPORTING COMPONENT (AT THE TAIL OF THE ARROW)
- INDICATES A MODIFYING QUANTITY

Fig. 13—Functional diagram for an illustrative single-axis controlled member intertial-space geometrical stabilization system. The diagram illustrates the functional relationships among the operating components forming a single-axis space angular integrating system.

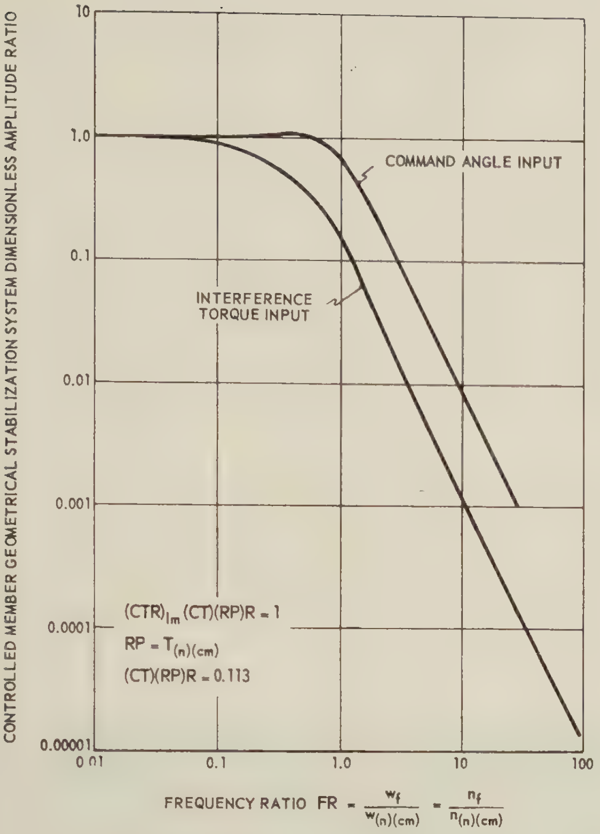


Fig. 14—Magnitude of frequency functions for a typical controlled member geometrical stabilization system.

type systems have demonstrated performance at a level that holds real promise for many purposes both civilian and military. As yet, operational experience with production equipment is just beginning, so it is impossible to give statistically valid results, even if the restrictions of security were removed. However, the fact that many manufacturers are either in production on inertial guidance systems or are competing for contracts is strong evidence that inertial devices are assuming greater importance in the search for accurate guidance to meet the stringent requirements of the space age.

The laws of nature that are applied in designing inertial-guidance equipment are all classical. No breakthroughs in science are involved. The key to inertial guidance is high performance for gyro units and specific force receivers. During the past few years, units of these types have been improved by several orders of magnitude as far as drift rates and uncertainty levels are concerned. These improvements have come from many sources, including design changes, better materials, better heat treatment, better gauging, better techniques, increased attention to cleanliness, and many other factors. The end to this improvement has certainly not yet arrived. Better components will surely be available as developments proceed. The result will be that inertial subsystems will be applied in many systems that also use other principles, to realize results far beyond any that are given by equipment now in common use. The series of developments that are to come will surely be very interesting and will provide stimulation for many engineers and scientists.

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Baseline Guidance Systems*

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Summary—This paper surveys the principles underlying the radio guidance and tracking of space vehicles. The various techniques which can be used for obtaining position and/or velocity information are described and their relative merits are assessed. The limitations imposed by tropospheric and ionospheric noise on radio systems are also discussed. Finally, two specific examples are worked out, illustrating the translation of radar guidance errors into flight uncertainties in the trajectories of ICBM's and satellites.

I. Introduction

THE ADVENT of the long-range ballistic rocket and its application to ICBM's, satellites, and the forthcoming lunar probes, have heightened interest in various kinds of tracking and guidance systems. The purpose of this paper is to review some of the general tracking and guidance concepts as exemplified by the use of baseline configurations in radio-guidance systems and, in so doing, to highlight some of the basic physical considerations, theoretical relations, and guidance system applications. The term "baseline" as used herein merely denotes the configuration of two radio receivers separated by a certain distance.

A number of different ways of obtaining and utilizing the information derived from such a simple system, or a complex one comprising several of these baselines, are described here. The treatment is restricted primarily to considerations which fix the position and velocity vectors of a space vehicle, and details concerning the means by which the vehicle is kept on a prescribed path or guided to a particular point in space are not given. Neither does the scope of the paper allow any treatment of the multiple-tracker problem. By this is meant the use of additional down-range tracking facilities to improve the determination of the vehicle trajectory by the insertion-guidance complex. Fig. 1 shows an artist's conception of the particular case of guiding a satellite into orbit using a down-range guidance system only. An approach such as this would be used to take advantage of the greater accuracy resulting from the increase in the elevation angle.

For the purposes of this discussion, the tracking concepts are broken into three basic categories:

1) systems which extract position as primary sensory information,
2) systems which extract velocity as primary sensory information,
3) systems which extract both position and velocity as primary information.

Many existing tracking systems do not fall directly into any of these formal categories because of their special applications. For example, in the tracking of satellites, the equations of motion in the earth's gravitational field (neglecting atmospheric effects for low perigee orbits) provide enough information so that range need not be measured directly to establish the orbit.

Since we are concerned here with guidance as well as tracking, the discussion will be confined to systems which provide real-time data to a guidance computer for calculation and generation of rocket control and other commands.

The special purpose computer must be regarded as an integral part of the guidance system since it is intermediate in the loop between the sensory data and the commands transmitted to the vehicle to correct its motion. Some further comments on the computer will be made in the next section.

Fig. 1—Artist's conception of the guidance of a satellite into orbit.

II. System Concepts

In the guidance of a ballistic rocket it is required that the vehicle's position and velocity be known continuously during the powered flight. From this information, appropriate steering commands can be calculated and transmitted to the vehicle so that it will follow a desired trajectory to cut-off. At cut-off the vehicle must have the proper position and velocity vectors as appropriate initial conditions for the ballistic trajectory, whether the purpose be to hit a certain target on the earth or moon, or to achieve a desired orbit around the earth, or to send a vehicle on an interplanetary flight.

The position and velocity can, in general, be provided by measuring:

1) position, and differentiating the position data to obtain the velocity components,
2) rates, and integrating them to obtain position,
3) both position and rate components.

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It is evident that many operational considerations must be weighed before selecting a system; however, this discussion will be restricted to technical matters alone.

A brief résumé of the role of the computer in the guidance loop is appropriate at this point. The computer is a special purpose device, programmed to solve a given set of guidance equations in real time. The primary purpose of this computer (but not by any means the only one) is to generate commands based upon a comparison of the radar input data and the desired flight performance. These commands may consist of pitch and yaw turning rates plus other discrete commands, such as those which shut off the motors of the various stages.

The turning commands continuously adjust the rocket's velocity vector so as to bring the three components thereof to the correct values corresponding to the selected position vector at cut-off of the final stage.

In treating the input data, the computer will perform averaging, or smoothing, processes and should even apply criteria for the rejection of spurious information. If the smoothing times available are sufficiently long, it may be feasible to obtain rate information by differentiating the position data instead of having a separate rate system. Of course, this alternative is contingent upon the accuracy requirements which the velocity data must fulfill.

At the end of each computation cycle, the turning rates and other control commands are coded and transmitted to the vehicle, via the data link. The message is received in the airborne receiver, decoded, and transmitted to the autopilot, which in turn actuates the gimballed engines or rocket vanes to effect the desired turns. The discrete control commands, which might be used to turn scientific instruments on or off, or to prearm a warhead, are also processed by the decoder and sent through the appropriate channels.

III. Measurements of Position

Position measurements with a baseline system can be obtained, generally, in three ways: angle measuring techniques, ranging techniques, and combinations of these.

A. Angular Measurements

While there are several variations possible in practice, all schemes can be understood fundamentally on the basis of the following simplified model. The system is essentially an interferometer, wherein there are two receivers separated by a distance $2D$ (see Fig. 2). For a distant target the phase difference of the waves arriving at the two stations is given by

$$\phi = (2\pi/\lambda)(R_1 - R_2) = (2\pi/\lambda)2D \sin \theta \qquad (1)$$

where λ is the wavelength of the radio wave, and θ is the angle (from the normal) of arrival. Measurement of ϕ thus yields θ. Note, however, that there is an ambiguity in the determination of θ since ϕ is cyclic for a change in $R_1 - R_2$ of an integral multiple of λ. Note also that greatest sensitivity is achieved around $\theta = 0°$, and poorest sensitivity, around $\theta = 90°$. With fixed antennas, this deterioration in sensitivity must be accepted; but for very short baseline systems, to be described later, it is always possible to work in the vicinity of $\theta = 0°$.

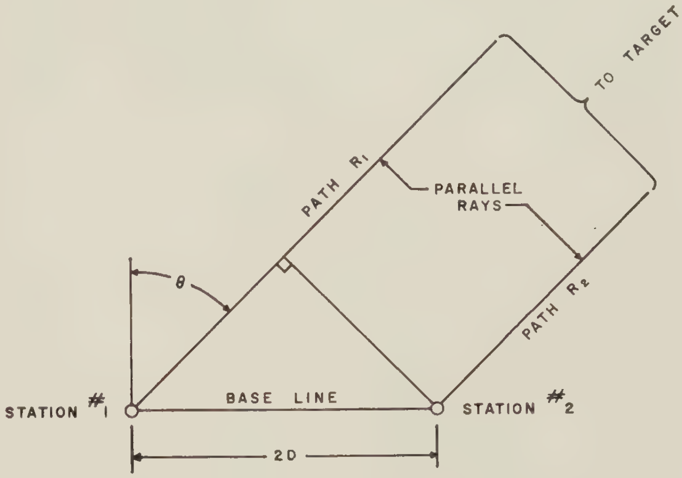

Fig. 2—A simple interferometer for measuring angular position of target.

To find the sensitivity of this interferometric system, take differentials on both sides of (1):

$$\delta\phi = \frac{2\pi}{\lambda} 2D \cos \theta \delta\theta$$

or

$$\delta\theta = \frac{\lambda}{2\pi} \cdot \frac{1}{2D \cos \theta} \delta\phi. \qquad (2)$$

Thus, for fixed operating frequency and baseline length, the accuracy with which angular position can be measured is set by the limitation on the measurement of phase. Conversely, as is often the case, if the angular accuracy and operating frequency are specified, the baseline length is then determined. For example, if 1-mr accuracy is required around the $\theta = 0°$ position for an operating wavelength of 1 foot ($f = 1000$ mc) and phase measuring capability of 5°, it is found that D must be about 15 feet. For X band ($\lambda \approx 0.1$ foot), D would be reduced to about 1.5 feet.

Examination of (1) shows that to achieve high accuracies, it is desirable to use short wavelengths and long baseline lengths. However, with increased sensitivity, the ambiguity intervals become more closely spaced. Resolution of these ambiguities may be effected by using either additional collinear baseline lengths or modulation techniques, or by keeping continuous track of the angles from an initially known set of values, if that is possible.

To obtain the range information by use of interferometer techniques, a second baseline is needed at a known position with respect to the first. For simplicity, we choose two perpendicular legs (see Fig. 3). A simple

analysis shows that in addition to the phase differences between the orthogonal pair of stations (*i.e.*, ϕ_{32} and ϕ_{31}), the phase difference between the pair 12 is also required. However, unless D is an appreciable fraction of R, ϕ_{12} will be only the average of ϕ_{32} and ϕ_{31}, and hence will yield no new information. For distant targets this limitation dictates a need for very long baselines to obtain R accurately and, therefore, a system of this simple type is never used for both angular and range determinations.

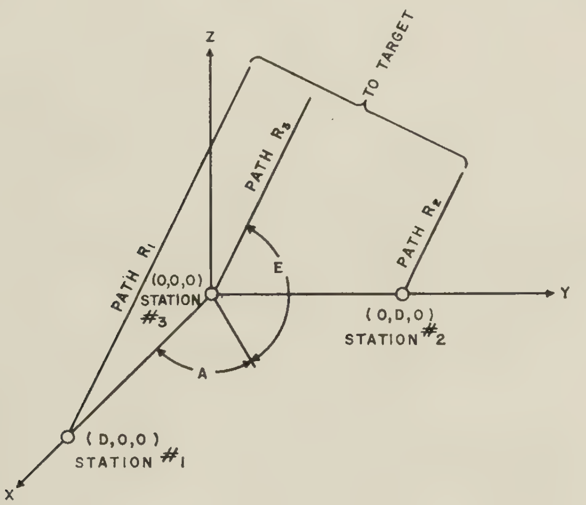

Fig. 3—Position determination from three range measurements.

B. Range Measurement Techniques

The use of three radars which measure range only suffices to determine completely the position of a target. This can be seen qualitatively by observing that the locus of possible positions as seen by each instrument is a sphere and that the intersection of the three spheres fixes the position uniquely. If the three stations are located as indicated in Fig. 3, and the central one is chosen as the origin of coordinates, then R is obtained by direct measurement from this central station while the elevation E and azimuth A angles are found by solving the range equations,

$$R_1^2 = R^2 - 2xD + D^2$$
$$R_2^2 = R^2 - 2yD + D^2$$
$$R^2 = x^2 - y^2 - z^2 \tag{3}$$

in conjunction with the relations

$$x = R \cos E \cos A$$
$$y = R \cos E \sin A$$
$$z = R \sin E. \tag{4}$$

Of interest is the uncertainty in the angular positions arising from errors in the range measurements. Considering the range errors in each of the radars to be uncorrelated but of equal mean squares, the rms error in the azimuth angle A is given by (assuming $D<R/10$, and setting $R'=R/D^2$)

$$\sigma_A = \frac{(2 - \sin 2A)^{1/2}}{|\cos E|}\, \sigma_{\delta R'}. \tag{5}$$

Similarly, for the rms error in the elevation angle

$$\sigma_E = \frac{(2 + \sin 2A)^{1/2}}{|\sin E|}\, \sigma_{\delta R'}. \tag{6}$$

Universal curves of $(2-\sin 2A)^{1/2}$ and $(2+\sin 2A)^{1/2}$ (which are merely shifted 90° apart in A) are shown in Fig. 4. Qualitatively, the curves tell us that azimuthal angles are measured most accurately at low elevation angles and that elevation angles are measured most accurately at large elevation angles. In considering a specific numerical example, present-day range accuracies may be taken to be of the order of 100 feet ($=\sigma_{\delta R}$). Assuming that accuracies better than 1 mr are required in elevation, for elevations above 10°, reference to the curves shows that for this case $\sigma_{\delta E}/\sigma_{\delta R'} \doteqdot 10$ or $\sigma_{\delta R'} = 10^{-4}$ and hence that $D = 10^6$ feet or about 200 miles. This is quite a long baseline and would have to be even longer if tracking had to be accomplished in both elevation and azimuth for low and high elevation angles.

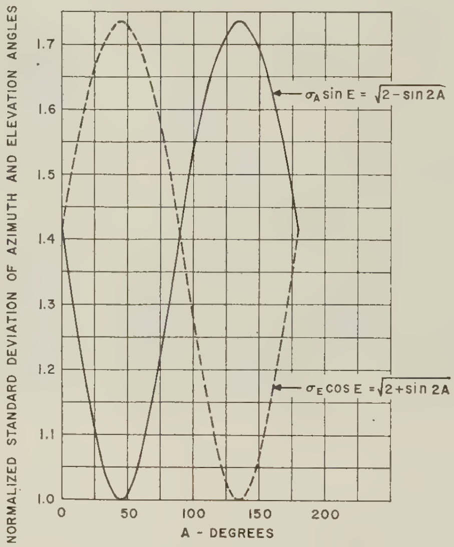

Fig. 4—Normalized standard deviation of azimuth and elevation angles vs azimuth angle.

C. Combination Angle and Range Measurements

In the preceding sections it has been shown that inordinately long baselines are required for the accurate determination of range from an angle-measuring scheme and for the accurate determination of angle from a range-only measuring complex. Merging the better features of both of these systems will result in an extremely compact radar capable of measuring both range and position angles accurately. The range is measured with pulses, and angles are measured on a short baseline, in the manner described in Section III-A.

The tracking function in this radar is achieved by use of a servo system controlled by the magnitude and sense of the phase difference given by (1). The servo

tends to rotate the receiving antennas shown in Fig. 1 about their common midpoint so as to keep θ equal to zero. Thus there is no deterioration of sensitivity with angle aspect, and the problem of ambiguities does not arise. This is, of course, a description of a conventional tracking radar.

Since angular positions can be so accurately measured with a long baseline system, it is natural to inquire if such a system could not be adapted to measuring range with high precision. In principle, this can be accomplished by suitably modulating the continuous wave used to provide the phase difference data and thence the angles. Without going into detail, the procedure involved amounts to putting "markers" on the radio signal which serve the same purpose as pulses in a range radar.

D. Engineering Problems

One of the primary problems of long baseline interferometer systems is the necessity for maintaining a very high degree of phase stability across the baseline, so that phase differences resulting from the target's position are not degraded by spurious phase errors. Temperature control of the transmission lines, the use of phase-locked ultrastable oscillators, and numerous other refinements help to produce and maintain the phase synchronizations necessary for precise phase measurements. Careful attention to these problems and those associated with the resolution of ambiguities mentioned previously is important in improving the operability of such systems. It is anticipated that high precision systems of the kind described above will continue to have a dominant role in the technology of tracking systems.

In a system which calculates rates from position information, it is usually necessary to provide an appreciable amount of data smoothing in order to remove the system noise. In addition, when long smoothing times are used, it is necessary to have a good analytical model for system data so that the data may be properly weighted and updated. This is a relatively straightforward procedure, in tracking a satellite, where the Keplerian relations provide the needed model; however, for rockets under high accelerations in powered flight, the establishment of an adequate model is a more difficult matter.

IV. Velocity Measuring Systems

In systems which measure velocities and calculate positions by integration, it is necessary to know the position at the starting point and to be assured that continuous rate information will be obtained. These are Doppler systems in which Doppler frequencies and differences thereof are used to calculate range rates and angular rates. Depending upon the coordinate system selected, this procedure may or may not be direct. For example, for a system which measures $\dot{R}$, $\dot{E}$, and $\dot{A}$ (range rate, angular elevation rate, and azimuthal rate, respectively), only $\dot{R}$ can be obtained directly. Referring to Fig. 2, range rate differences are given by

$$\dot{R}_3 - \dot{R}_1 = 2D(\sin E \cos A\dot{E} + \cos E \sin A\dot{A})$$
$$\dot{R}_2 - \dot{R}_4 = 2D(\sin E \sin A\dot{E} - \cos E \cos A\dot{A}). \quad (7)$$

Hence, to solve for $\dot{E}$ and $\dot{A}$, E and A must be assumed known. Thus the extraction of angles becomes an iterative process in which it is necessary to know the angle in order to compute the angular rate, and hence to generate an incrementally different angle for the succeeding calculation.

A system such as this requires initial position information, making the acquisition of targets difficult. Furthermore, a temporary loss or degradation of signal due to a poor antenna look-angle, exhaust attenuation, or other cause, requires a prediction of position changes. Depending upon the specific system function, this may or may not be a serious consideration.

A variety of techniques may be used for velocity measuring systems. Eq. (2) shows that if the rate of change of phase is measured between two stations, the angular rate may be deduced. The use of a radar tracker provides the necessary position information to solve (7) for $\dot{E}$ and $\dot{A}$. Operationally, a CW transmitter whose signal triggers a beacon on the vehicle serves also as the reference for the signal which is synchronously amplified, offset, and transmitted to the ground. The use of an ultrastable airborne oscillator would greatly simplify the frequency synchronization problem. A CW signal would not have to be transmitted to the vehicle, since the Doppler shift in the frequency received at the ground from the known frequency transmitted by the oscillator would suffice to give the radial and angular velocity components of the vehicle. The utilization of new atomic frequency standards with stabilities in the order of $1:10^{10}$ shows promise of providing a relatively simple and straightforward technique for getting accurate velocity information.

An inherent advantage of Doppler rate systems is their ability to provide real-time rate information without need for long time smoothing. However, an adequate model for the vehicle's behavior must be available for filling in lapses of information due to signal interruption.

As an example of the kind of frequency stability and baseline length required for the accurate measurement of rates, consider a satellite at 300 miles altitude ($v \doteq 25{,}000$ fps). Under the most favorable circumstances, this could be the radial rate, and the round-trip Doppler frequency observed for a wavelength of 0.1 foot ($f = 10^{10}$ cps) would be

$$f_0 = \frac{2\dot{R}}{\lambda} = 500 \text{ kc}.$$

If we should like to measure $\dot{R}$ within 5 fps, *i.e.*, 0.02 per cent, then we must measure f_0 to the same precision,

i.e., 100 cps. Thus, in this case, the frequency stability must be held to 1 part in 10^8. The maximum angular rate will occur when the satellite passes directly overhead, and its value is then about 1.4×10^{-2} radians per second. A slight modification of (1) and (2) gives us

$$\dot{R}_1 - \dot{R}_2 = D \cos \theta \dot{\theta}$$

where R_1 and R_2 are the distances from the satellite to the two stations. To measure $\dot{\theta}$ with a 1 per cent accuracy around $\theta = 0$ requires, therefore, that the range-rate difference be measured within $1.4 \times 10^{-4} D$. Hence, if the measuring limit on $\dot{R}_1 - \dot{R}_2$ is 0.1 fps, D must be about 700 feet long.

V. Propagation Considerations

An important factor to be taken into account in the design of any radio guidance system is the amount of degradation in the position and velocity information induced in the measured data by the nature of the atmosphere. (The term "atmosphere" will include the ionosphere for very high radar targets.) Deterioration of the radio signals occurs because of the spatial inhomogeneity of the atmosphere and its variation in time. The space variations taken alone result in statical or bias errors, while the time-varying or fluctuating components lead to rms errors.

Several procedures are immediately suggested for minimizing these undesirable effects. These procedures will be outlined first and then treated in more detail. First, if measurements of the atmospheric parameters contributing to the radar uncertainties are made just prior to the time of interest, the statical errors can be fairly reliably calculated and hence compensated. Second, insofar as ionospheric phenomena are concerned, adverse radar effects can be ameliorated by the use of higher frequencies. Third, inspection of (2) shows that fluctuations in angular positions are decreased by using a larger baseline length D. And finally, the fluctuating components can be handled by smoothing the data over a period of time comparable with the fluctuation periods.

A. Tropospheric Effects

The determination of bias effects mentioned above is relatively straightforward. In the troposphere, the radar index of refraction, valid for frequencies at least up to 10 kmc, is given by

$$(n - 1)10^6 = \frac{77.6}{T}\left(p + \frac{4810}{T} e_s RH\right) = N, \text{ say,} \quad (8)$$

where T = temperature (degrees Kelvin), p = pressure (millibars), e_s = saturation vapor pressure of water (mb), and RH = relative humidity.

Thus, knowledge of the above parameters as a function of altitude gives an N profile. (This type of information is obtainable from radiosonde data.) The refraction error is given by

$$\Delta E = \cot E \frac{1}{H} \int_0^H (n_0 - n) dh \text{ (radians)} \quad (9)$$

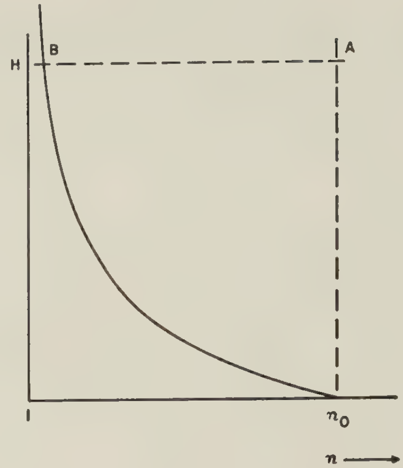

Fig. 5—Nominal index of refraction vs height.

or

$$\Delta E = \cot E (n_0 - \bar{n}) \quad (10)$$

where E = the elevation angle, n_0 = the index at the radar level, n = the index at height h, H = the target height, and $\bar{n}$ = the average index up to height H.

Eq. (11) is based on a horizontally stratified atmosphere above a flat earth. (The modification to the curved earth is minor.) The nominal behavior of n with height is shown in Fig. 5.

For high altitudes, (12) simplifies to

$$\Delta E \doteqdot N_0 \cot E \text{ microradians} \quad (11)$$

Values of N_0 vary from 250 to 400, so that at a 10° elevation, say, ΔE varies from 1.4 to 2.3 milliradians. However, N_0 at a given site and a given time can be measured quite accurately at the ground so that the uncertainty in the correction ΔE can be reduced to a small fraction of a milliradian.

Similarly the range error is given by

$$\Delta R = \csc E \int_0^H (n - 1) dh = R\overline{N}. \quad (12)$$

Since N is approximately zero above about 100,000 feet, the integration in (13) can be cut off there. Radiosonde data show that up to 100,000 feet, N is about 90. Hence the range error, looking vertically, is

$$\Delta R = 100{,}000 \times 95 \times 10^{-6} \doteqdot 9 \text{ feet.}$$

This amount increases as the secant of the depression angle from the vertical. The error does not exceed 50 feet until this angle reaches 80° (E = 10°). Radiosonde data taken prior to launch would give an accurate estimate of $\bar{n} - 1$ and the residual error, after the correction is made, should be a small fraction of the value given above.

For the horizontally stratified atmosphere assumed above, there is no azimuthal refraction error. In practice, however, horizontal gradients in temperature, pressure, and humidity can occur, and will give rise to an azimuth error. Experimental programs have shown that, in the main, these bias effects are of relatively short duration and that the important effects in the measure-

ment of azimuth are those due to atmospheric fluctuations. These are treated in Section V-C.

B. Ionospheric Effects

The index of refraction for radio waves in the ionosphere is given by

$$n^2 = 1 - \frac{N_e e^2}{4\pi^2 \epsilon_0 m f^2}$$
$$= 1 - \frac{80 N_e}{f^2} \qquad (13)$$

where N_e = electron density (meter^{-3}), and f = frequency of the radio wave.

Range and angular refraction errors may now be calculated in the same manner as for the troposphere, using (13) for the index. Assumption of a resonable model for the ionosphere results in a refraction of from 5 to 10 mr for a ray from a target at 20° and about 300 km in height, for a frequency of 100 mc. At 1000 mc, the values are reduced by the square of the frequency ratio, *i.e.*, 100-fold. Such values, then, are small compared with other sources of error. The range error looking vertically through the entire ionosphere (based on data obtained from moon-echo experiments) is less than 60 feet at 100 mc. The error is down to 0.6 foot at 1000 mc.

The above description of ionospheric effects has been greatly simplified. At lower frequencies ($\leq$100 mc), the effect of the earth's magnetic field on electromagnetic waves cannot be neglected. The ionosphere becomes a more strongly doubly-refracting medium with the net result that the plane of polarization of the wave is rotated (the Faraday effect). Antennas have to be designed to accept circularly polarized waves if this effect is important.

While ionospheric effects and baseline accuracies call for as high an operating frequency as possible, an upper limit is imposed by the amount of signal attenuation tolerable. Absorption due to the moisture content of the atmosphere increases with frequency. A 10-kmc path through a heavy rain would produce about 6 db of attenuation at a frequency of 10 kmc, while at 30 kmc the loss is about 55 db. Thus the desire for all-weather operational capability sets an upper limit of about 10 kmc for the frequency.

C. Fluctuating Effects

Time variations in the index of refraction, while they may not contribute to the bias errors described above, will lead to mean-square errors and as such be combined with other independently arising errors. The physical basis for this type of noise is to be found in the dynamic character of the atmosphere. Fluctuations of pressure, temperature, and humidity, particularly those associated with clouds, are the contributing causes in the troposphere, while electron density variations are the causes in the ionosphere. Since the errors induced in range and range rate are negligible for any present systems, only angular positions and rates are considered.

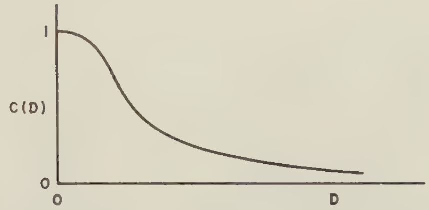

Fig. 6—Sketch of a nominal correlation function.

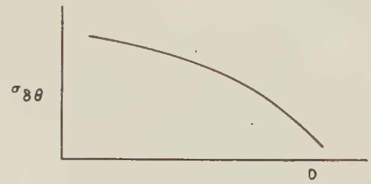

Fig. 7—Plot of standard deviation of angular position vs baseline length.

From (2), an apparent angular shift in position will occur due to a shift in the phase difference of the rays arriving at two receivers from a distant source. If the spurious phase additions added to each wave are the same, they will cancel out in the subtraction process. In general, while these phases will not be the same, they will be correlated to a degree depending on the displacement of the two rays. Mathematically, the mean square phase difference between the stations $\overline{\delta\phi^2}$ is related to the mean square phase variation $\overline{\delta\phi_i^2}$ at a given station by the relationship

$$\overline{\delta\phi^2} = 2\overline{\delta\phi_i^2}[1 - C(D)] \qquad (14)$$

where $C(D)$ is the so-called correlation function and has the general shape shown in Fig. 6. The expression for the rms angular fluctuation then becomes

$$\sigma_{\delta\theta} \equiv \sqrt{\overline{\delta\theta^2}} = \frac{\lambda}{2\pi D} \sqrt{2\overline{\delta\phi_i^2}}[1 - C(D)]^{1/2}. \qquad (15)$$

The effect of the bracketed square-root term is to make the rate of decrease of error with increasing baseline length somewhat less than that due to the D^{-1} factor. This D^{-1} type of decrease is obtained only for large values of D. The general behavior of $\sigma_{\delta\theta}$ with baseline length is shown in a log-log plot in Fig. 7.

On this basis, $\sigma_{\delta\theta}$ can be indefinitely reduced by choosing larger D's. In practice, a value in the hundreds of feet will reduce $\sigma_{\delta\theta}$ to hundredths of a milliradian.

The fluctuations in $\dot{\theta}$ can be treated similarly. The relationship between $\sigma_{\delta\dot{\theta}}$ and D is of the same nature as that shown in Fig. 7. In both cases, further reduction in the rms values can be achieved by lengthening the smoothing period.

Within the scope of this paper we cannot discuss further the many additional propagation factors which must be considered in the design of a tracking and guidance radio system. The use of high frequencies (in the kilomegacycle region) obviate the necessity, for present accuracy considerations at least, of examining in detail the effects due to auroral activity, meteors, and other miscellaneous phenomena which can seriously affect lower frequency radio propagation.

VI. Orbital Insertion Accuracies

In this section, the accuracies with which a long baseline guidance system can insert a space vehicle into orbit are analyzed. The problem requires that the orbital insertion parameters ρ_0, $|\vec{V}|$, and β (distance from earth center, speed, and angle the velocity vector makes with the local vertical, respectively) be expressed in terms of the radar observables (see Fig. 8). Then variations in members of the former set of variables can be written in terms of the standard deviations of members of the second set. As members of this second set we take range, elevation angle, and their time derivatives. The assumption that the radar system lies in the orbital plane allows azimuth errors to be neglected.

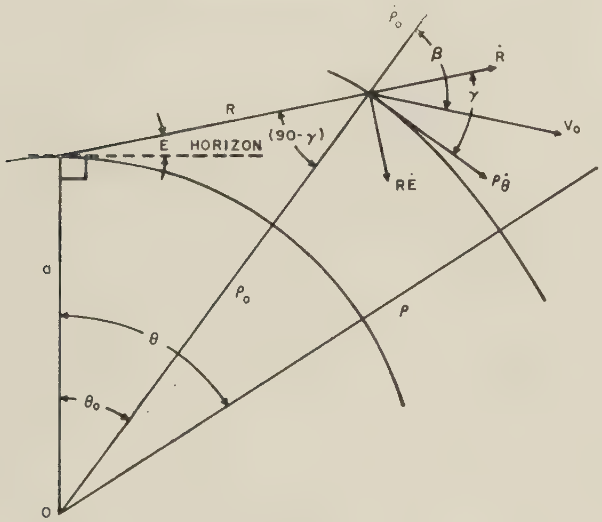

Fig. 8—Geometry of insertion of a space vehicle into orbit.

Two specific examples will be considered, using hypothetical accuracies for the radar guidance system. These numbers do not represent the present state of the art, nor do they refer to any actual system. We shall assume that angular measurements can be made to within an accuracy of 1 mr (1 mr$=0.0573°$), range measurements within 100 feet, angular rates within 10 μr/second and range rate within 10 feet per second. We have assumed a baseline about 5500 feet long so that the elevation angle chosen, *viz.*, 10°, results in an effective baseline 1000 feet long Modifying (2) to the form

$$D_{\text{eff}}\delta\dot{\theta} = \frac{\lambda}{360}\,\delta\dot{\phi}°,$$

and assuming that $\dot{\phi}$ can be measured to 36°/second, results in the value of 10μr/second for $\delta\dot{\theta}$ for $\lambda=0.1$ foot. These data are summarized in Table I.

TABLE I

Radar Observable	Standard Deviation
Range—R	R—250 feet
Elevation Angle—E	E—1 mr
Range Rate—$\dot{R}$	$\dot{R}$—10 feet per second
Elevation Rate—$\dot{E}$	$\dot{E}$—10 μr/second

Consider first the case relating to the insertion of a satellite into a 300 statute mile circular orbit. Choosing the range at insertion to be 1000 miles, the elevation angle turns out to be not quite 11°. The standard deviations of the orbital insertion parameters are derived in Appendix I and are stated simply:

$$\sigma_{\rho_0} = \frac{1}{\rho_0}\left\{\sigma_R^2(R + a\sin E)^2 + \sigma_E^2(aR\cos E)^2\right\}^{1/2} \qquad (16)$$

$$\sigma_{V_0} = \frac{1}{V_0}\left\{\sigma_{\dot{R}}^2\dot{R}^2 + \sigma_R^2(R\dot{E}^2)^2 + \sigma_{\dot{E}}^2(R^2\dot{E})^2\right\}^{1/2} \qquad (17)$$

$$\sigma_\beta = \frac{1}{V_0^2}\Bigg\{\sigma_{\dot{R}}^2(R\dot{E}^2) + \sigma_R^2\left(\dot{E}\dot{R} + \frac{V^2\cos\gamma}{a\cos\theta + R\sin\gamma}\right)^2 + (R\dot{R})^2\sigma_{\dot{E}}^2 + \left(\frac{V^2 a\cos\theta}{a\cos\theta + R\sin\gamma}\right)^2\sigma_E^2\Bigg\}^{1/2}. \qquad (18)$$

The values to be used for $\dot{R}$ and $\dot{E}$ in (16)–(18) follow from the geometry of the assumed initial conditions. The circular velocity appropriate to the 300-mile altitude is about 24,800 feet per second and this, with the range of 1000 miles, determines $\dot{R}=24{,}400$ fps and $\dot{E}=2.06\times10^{-4}$ rad/second. The earth central angle θ turns out to be about 13°, making $\gamma(=E+\theta)$ about 24°. These constitute the values of all the quantities needed for substitution into the above equations. The results obtained after performing the substitutions are

$$\sigma_{\rho_0} = 4800 \text{ feet}$$
$$\sigma_{V_0} = 10 \text{ feet per second}$$
$$\sigma_\beta = 2.3 \text{ mr} = 0.13°.$$

The numerical work shows that all of these results are quite insensitive to the value $\sigma_R=250$ feet. This latter value can be increased by more than an order of magnitude before it has any effect on the insertion errors.

To see what the magnitudes of the insertion errors mean in terms of the subsequent orbit, Table II gives

TABLE II

Partial Derivative	Value	Maximum Value of $\delta\rho$	
		Algebraic	Numerical
$\partial\rho/\partial\rho_0$	$1+2\sin^2\theta/2$	$3\delta\rho_0$	2.7 statute miles
$\partial\rho/\partial V_0$	$4\rho_0\sin^2\theta/2/V_0$	$4\rho_0\delta V/V_0$	7 statute miles
$\partial\rho/\partial\beta$	$-\rho_0\sin\theta$	$\rho_0\delta\beta$	10 statute miles

the partial derivatives of the radius vector for a nearly circular orbit in terms of the central angle and of the three initial parameters already mentioned. For a derivation of the related equations, see Appendix II. The maximum variation of the radius vector is then given for the numerical values listed above.

Some present-day systems are capable of doing even better than these figures indicate.

Our second example deals with the guidance of an

ICBM into orbit. Again referring to Fig. 8, the same elevation angle at insertion is chosen as for the previous case, *viz.*, 11°. The insertion height of 300 miles is somewhat high, so a lesser value is chosen of about 190 miles or, more precisely, 10^6 feet. This results in a more reasonable velocity value, as is shown below.

The tangential velocity $\rho_0\dot{\theta}_0$, if taken as 21,900 feet per second, yields the simple value of 10^{-3} for $\dot{\theta}_0$. If the impact point is taken to be 90° away from insertion, a relationship between $\dot{\rho}_0$ and $\rho_0\dot{\theta}_0$ is fixed (see Appendix III). This equation gives $\dot{\rho}_0 = 6.24 \times 10^3$ feet per second. The rest of the parameters are then determined by the geometry of the problem and are given for reference below:

$$
\begin{aligned}
a &= 2.09 \times 10^7 \text{ feet}\\
\rho_0 &= 2.19 \times 10^7 \text{ feet}\\
h &= 1.0 \times 10^6 \text{ feet}\\
\dot{\theta}_0 &= 10^{-3} \text{ rad/second}\\
\dot{\rho}_0 &= 6.2 \times 10^3 \text{ feet per second}\\
R &= 3.59 \times 10^2 \text{ feet}\\
V &= 2.28 \times 10^4 \text{ feet per second}\\
\dot{R} &= 1.62 \times 10^3 \text{ feet per second}\\
E &= 10°45'\\
\dot{E} &= 6.35 \times 10^{-3} \text{ rad/second}\\
\beta &= 74°06'\\
\theta &= 9°15'\\
\gamma &= 20°.
\end{aligned}
$$

The resulting standard deviations of the insertion parameters are

$$
\begin{aligned}
\sigma_{\rho_0} &= 34 \text{ feet}\\
\sigma_{V_0} &= 36 \text{ feet per second}\\
\sigma_\beta &= 1.0 \text{ mr.}
\end{aligned}
$$

The important "miss coefficient" for an ICBM is the dispersion in miles at the target due to the initial errors. Table III presents these numbers for the particular case chosen (see Appendix III for the derivations) and the down-range misses resulting when the above numerical values for the insertion dispersions are used. The symbol X is used for the down-range distance and for this case equals $\pi a/2$.

TABLE III

Partial Derivative	Value	Down-Range Miss
$\partial X/\partial \rho_0$	6.7 nautical miles/nautical miles	3.8 nautical miles
$\partial X/\partial \beta$	−3.4 nautical miles/mr	3.4 nautical miles
$\partial X/\partial V_0$	1.21 nautical miles/fps	43.5 nautical miles

The miss resulting from the velocity error is thus the dominant contribution to the total error. Examination of (17) reveals that for the values of the radar errors here taken, σ_{V_0} is very accurately given by one term only, *viz.*,

$$\sigma_{V_0} \doteqdot \frac{1}{V_0}\left[R^2\dot{E}\sigma_{\dot{E}}\right]. \tag{19}$$

Hence, to reduce the error resulting from this term, the bracketed quantity should be reduced. This is what is done in practice; security restrictions preclude our giving further specific information on this subject.

The foregoing section has presented an idea of the kinds of radar accuracies required to perform two of the new space-age missions. As requirements for accuracy in guidance become more stringent, longer baseliness will be used. To be sure, this technique poses some difficult problems, but it does possess the distinct advantage of throwing much of the burden on the geometry of the system rather than on the components. For this reason, long baseline guidance systems appear to offer distinct advantages over other guidance techniques. In principle, extremely long baselines stretching from the earth to an artificial satellite or even to the moon are feasible and will offer exciting possibilities for the guidance of interplanetary vehicles.

Appendix I

Derivation of Orbital Insertion Errors in Terms of Radar Errors

Referring to Fig. 8,

$$\rho_0^2 = R^2 + a^2 + 2aR\sin E. \tag{20}$$

Taking differentials yields

$$\delta\rho_0 = \frac{1}{\rho_0}\left\{\delta R(R + a\sin E) + \delta E aR\cos E\right\} \tag{21}$$

whence

$$\sigma_{\rho_0} = \frac{1}{\rho_0}\left\{\sigma_R^2(R + a\sin E)^2 + \sigma_E^2(aR\cos E)^2\right\}^{1/2} \tag{22}$$

which is (16) in the text.

The velocity error is easily derived from the simple relation

$$V_0^2 = \dot{R}^2 + (R\dot{E})^2 \tag{23}$$

by taking differentials:

$$\delta V_0 = \frac{1}{V_0}\left\{\dot{R}\delta\dot{R} + R\dot{E}^2\delta R + R^2\dot{E}\delta\dot{E}\right\}. \tag{24}$$

This equation gives, immediately:

$$\sigma_{V_0} = \frac{1}{V_0}\left\{\sigma_{\dot{R}}^2\dot{R}^2 + \sigma_R^2(R\dot{E}^2)^2 + \sigma_{\dot{E}}^2(R^2\dot{E})^2\right\}^{1/2}. \tag{25}$$

The standard deviation in the angle, β, of the velocity vector is slightly more difficult to come by, but is also straightforward. From Fig. 8,

$$\tan\beta = \rho_0\dot{\theta}_0/\dot{\rho}_0$$

or

$$\tan \beta = \frac{\dot{R} \cos \gamma + R\dot{E} \sin \gamma}{\dot{R} \sin \gamma - R\dot{E} \cos \gamma} \cdot \tag{26}$$

Differentiation of both sides of this expression and simplification leads to

$$D^2 \sec^2 \beta \delta\beta = - \delta\dot{R}R\dot{E} + \delta R\dot{E}\dot{R} + \delta\dot{E}R\dot{R} - \delta\gamma V^2 \tag{27}$$

where D is simply the denominator of the right-hand side of (26), *viz.*, $\dot{R} \sin \gamma - R\dot{E} \cos \gamma$. Taking the nominally circular orbit, $\beta \approx 90°$, and $\sec^2 \beta$ becomes very large. But, noting that

$$D \sec \beta = \frac{\dot{R} \sin \gamma - \dot{E} \cos \gamma}{\cos \beta} = \frac{\dot{\rho}}{\cos \beta} \left| \overrightarrow{V} \right| , \tag{28}$$

then $\delta\beta$ reduces to $1/V^2$ times the right-hand side of (27). But in this equation, $\delta\gamma$ must still be expressed in terms of differentials of the independent quantities. This is most readily done from the geometry of the problem. Some manipulation gives

$$\delta\gamma = \frac{\delta Ea \cos \theta + \delta R \cos \gamma}{a \cos \theta + R \sin \gamma} \cdot \tag{29}$$

Substituting this expression into (27) and taking account of (28), we obtain

$$\sigma_\beta = \frac{1}{V_0^2} \left\{ \sigma_{\dot{R}}^2 (R\dot{E})^2 + \left(ER + \frac{V^2 \cos \gamma}{a \cos \theta + R \sin \gamma} \right)^2 \sigma_R^2 + (R\dot{R})^2 \sigma_{\dot{E}}^2 + \left(\frac{V^2 a \cos \theta}{a \cos \theta + R \sin \gamma} \right)^2 \sigma_E^2 \right\}^{1/2} \tag{30}$$

which is (18).

Appendix II

Effect of Insertions Errors on a Satellite Orbit

The equation of the Keplerian ellipse can be written as

$$\frac{1}{\rho} = \frac{1}{\rho_0} \left\{ 1 + 2 \sin^2 \frac{\theta}{2} \left(\frac{\mu}{\rho_0 V_0^2 \sin^2 \beta} - 1 \right) - \sin \theta \cot \beta \right\} \tag{31}$$

where ρ is the radius vector, ρ_0 is the initial radius vector, μ is $GM(=1.4024 \times 10^{16}$ ft^3 sec$^{-2})$, V_0 is the initial speed, and the other terms have their previously defined meanings. Differentiating (31) with respect to ρ_0 and substituting $\beta = \pi/2$, $\rho_0 V_0^2 = \mu$, which hold for the circular orbit, we find that

$$\left. \frac{\partial \rho}{\partial \rho_0} \right|_{\beta=\pi/2} = 1 + 2 \sin^2 \frac{\theta}{2} \tag{32}$$

as listed in Table II.

Similarly, by differentiating with respect to V_0, we find that

$$\left. \frac{\partial \rho}{\partial V_0} \right|_{\beta=\pi/2} = 4\rho_0 \sin^2 \frac{\theta}{2} \Big/ V_0. \tag{33}$$

Also, differentiating with respect to β gives

$$\left. \frac{\partial \rho}{\partial \beta} \right|_{\beta=\pi/2} = - \rho_0 \sin \theta. \tag{34}$$

These results are also used in Table II.

Appendix III

Impact Errors of an ICBM Due to Insertion Errors

Taking the nominal impact point to occur for $\theta = 90°$ (see Fig. 8), the analysis is considerably simplified. Range errors δX will then be obtained from $\delta X = a\delta\theta$, where a = earth's radius. This assumption corresponds to a range of 6200 statute miles (5400 nautical miles) from the burnout position of the last stage. For this case, (31) is more conveniently recast into the equivalent form

$$\frac{1}{\rho} = \frac{\mu}{\rho_0^2 V_0^2 \sin^2 \beta} - \frac{\cos \beta \sin \theta}{\rho_0 \sin \beta} + \left(\frac{1}{\rho_0} + \frac{\mu}{\rho_0^2 V_0 \sin^2 \beta} \right) \cos \theta. \tag{35}$$

At impact, $\rho = a$, and we have chosen θ at that time to be 90°. Thus

$$\frac{1}{a} = \frac{\mu}{h^2} - \frac{\rho_0}{h} \tag{36}$$

where $h = \rho_0 V_0 \sin \beta$, the angular momentum.

For assumed values of ρ_0 and $\dot{\theta}_0$ (and therefore h), (36) determines $\dot{\rho}_0$. This is what was done in the example discussed above. For $a = 2.09 \times 10^7$ feet, $\rho_0 = 2.19 \times 10^7$ feet, $\dot{\rho}_0 = 10^{-3}$ rad/second, (36) gives $\dot{\rho}_0 = 6.24 \times 10^3$ fps. Also determined thereby is β, from $\beta = \tan^{-1}(\rho_0 \dot{\theta}_0 / \dot{\rho}_0) = 74.1°$.

Taking partial derivatives of (35), we obtain first, after a little manipulation

$$\frac{\partial \theta}{\partial V_0} = \frac{1}{V_0} \frac{2\mu}{\mu - hV_t} \tag{37}$$

where $V_t = \rho_0 \dot{\theta}_0 = V_0 \sin \beta$ is the initial tangential velocity. Secondly, we get

$$\frac{\partial \theta}{\partial \beta} = \frac{1}{V_t} \left(V_0^2 - \frac{2\mu \cot \beta}{\rho_0} \right) \frac{h}{hV_t - \mu} \tag{38}$$

and finally

$$\frac{\partial \theta}{\partial \rho_0} = \frac{V_0 V_t}{hV_t - \mu} \left(\frac{2\mu}{V_0 h} - \cos \beta \right). \tag{39}$$

Substituting the numerical values for the velocity components and other terms assumed above and multiplying by the radius of the earth gives the miss coefficients listed in the second column of Table III.

Contributors

Charles S. Draper (A'53–F'55) was born in Windsor, Mo., on October 2, 1901. He entered the University of Missouri in 1917, transferred to Stanford University in 1919, and received the B.A. degree in psychology in 1922. Later that year, he began his present association with the Massachusetts Institute of Technology, from which he received three degrees: the B.S. in electrochemical engineering in 1926, the S.M. in 1928, and the Sc.D. in physics in 1938.

C. S. Draper

He is presently Professor and Head of the Department of Aeronautical Engineering, and Director of the Instrumentation Laboratory at M.I.T. He has worked in the fields of aeronautical power plants, flight testing, vibration measurements, aeronautical instruments, and control engineering, with special attention to gyroscopic principles for military and commercial equipment. During the past fifteen years, his research and development efforts have been principally concerned with fire control, flight control, and inertial guidance systems for the Air Force and Navy. He is a member of several advisory groups connected with the military services.

Dr. Draper has written extensively in the fields of instrumentation and control. He has served as consulting engineer to many aeronautical companies and instrument manufacturers, and holds a number of patents for measuring and control equipment.

Recently he received world-wide recognition for his work in the field of inertial navigation for manned aircraft, missiles, and naval vessels. Design, construction, and testing of complete systems, as well as the development of high-performance components were included in this work.

Dr. Draper is a Fellow of the American Physical Society, the Institute of the Aeronautical Sciences, the American Academy of Arts and Sciences, the American Society of Mechanical Engineers, and the American Association for the Advancement of Science. He is a member of the National Academy of Sciences, the American Institute of Consulting Engineers, the Society of Automotive Engineers, the American Ordnance Association, the American Society for Engineering Education, the Massachusetts Society of Professional Engineers, the New York Academy of Sciences, Sigma Xi, Tau Beta Pi, and Sigma Alpha Epsilon.

He has received the following honors: Medal for Merit, Naval Ordnance Development Award, and Sylvanus Albert Reed Award of the Institute of the Aeronautical Sciences in 1946; New England Award of the Engineering Societies of New England in 1947; Exceptional Civilian Service Award of the Department of the Air Force, and a Testimonial of Appreciation from the Industrial Instruments and Regulators Division of the American Society of Mechanical Engineers in 1951. He was selected as Wilbur Wright Memorial Lecturer before the Royal Aeronautical Society in London in 1955. The following year he received the Navy Distinguished Public Service Award, and in 1957 he was presented with the Airpower Award of the Massachusetts Wing of the Air Force Association, the Thurlow Award of the Institute of Navigation, the Holley Medal of the American Society of Mechanical Engineers, and the Airpower Trophy of the Air Force Association. In 1958 he received the Blandy Medal from the American Ordnance Association.

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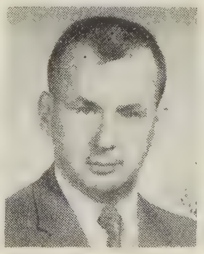
Robert S. Grisetti (S'49–A'50–M'55) was born in San Diego, Calif., on November 16, 1924. He attended the University of California at Berkeley and received the B.S.E.E. degree in 1948, and the M.S.E.E. degree in 1950.



R. S. Grisetti

In 1949 he entered the Advanced Engineering Program of the General Electric Company at Syracuse, N. Y., and three years later was made a program supervisor, a position he held for one year He then joined an advanced engineering group, working on such projects as the development of battlefield surveillance radar, VT fuses, and sonar equipment.

He is presently supervisor of the Systems Analysis Unit of the Missile Guidance Section and has for the past several years been associated with the company's Atlas guidance program.

Mr. Grisetti is a member of Tau Beta Pi and Sigma Xi.

❖

Earle B. Mullen (A'52–M'55) was born in Toronto, Canada, on March 3, 1925. He graduated from the University of Toronto in 1946 with the Bachelor of Arts degree in mathematics and physics, and two years later was awarded the Master of Arts degree in physics.

E. B. Mullen

After some experience in geophysical work and teaching, he joined General Electric's Electronics Laboratory, Syracuse, N. Y., and worked on a variety of problems in microwave propagation, particularly in ferrites. More recently, he has been associated with the Missile Guidance Section working on a variety of propagation problems and space exploration programs.

Mr. Mullen is a member of the ARS.

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H. Guyford Stever was born on October 24, 1916. He was graduated from Colgate University, Hamilton, N. Y., in 1938 and in 1941 received the doctorate degree in physics at California Institute of Technology, Pasadena, where he did research on cosmic rays and electronics for geiger counter experiments.

Dr. Stever then came to M.I.T. as a staff member of the Radiation Laboratory, where

he worked on the development of radar. From 1942 to 1945, he was scientific liaison officer in London representing NDRC in work on radar and guided missiles. In addition to his field work with the armed forces, he was a member of various technical intelligence missions in France, the Netherlands, and Germany. In recognition of his scientific contributions to the war effort, he was awarded the President's Certificate of Merit in 1948.

H. G. STEVER

Dr. Stever returned to the faculty of M.I.T. after the war, and became professor of aeronautical engineering and Associate Dean of the School of Engineering in 1956. In addition to his active role in teaching and administration at M.I.T., he has rendered service to the United States Air Force on several advisory groups, culminating in his service from 1955 to 1956 as chief scientist of the Air Force. For this service, he was given the USAF Exceptional Civilian Service Award.

Since the war, Dr. Stever has been a member of many scientific committees concerned with guided missiles and problems of air defense. His research in physics and aeronautical engineering has been in the fields of cosmic rays, radar, guided missiles, high-speed aerodynamics, high-speed flight, and nuclear powered aircraft.

Recently, he has been appointed Chairman of an NACA committee on space technology. A member of Phi Beta Kappa, Sigma Xi, Sigma Gamma Tau, Tau Beta Pi, he is also a Fellow of the American Academy of Arts and Sciences, the American Association for the Advancement of Science, the American Physical Society, and the Institute of Aeronautical Sciences.

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Peter Swerling (M'56) was born in New York, N. Y., on March 4, 1929. He received the B.S. degree in mathematics from California Institute of Technology, Pasadena, in 1947, the B.A. degrees in economics from Cornell University, Ithaca, N. Y. in 1949, and the M.A. and Ph.D. degree in mathematics from the University of California at Los Angeles in 1951 and 1955.

Dr. Swerling worked on Project RAND at Douglas Aircraft Company in 1947 and 1948. From September, 1949 to January, 1950, he was a teaching assistant in mathematics at U.C.L.A. In 1949 he was employed by RAND Corporation, working full time there in 1952. From 1956–1957 he served as visiting research assistant professor at the Control Systems Laboratory of the University of Illinois, before returning to RAND Corporation where he is presently employed.

P. SWERLING

Theory of random noise, especially as applied to radar performance, has been Dr. Swerling's major field of research.

He is a member of Phi Beta Kappa, Sigma Xi, Pi Mu Epsilon, the American Mathematical Society, and the Society for Industrial and Applied Mathematics.

CALL FOR PAPERS

The Third National Convention on Military Electronics, sponsored by the PGMIL, will be held on June 29–30–July 1, 1959, at the Sheraton Park Hotel, Washington, D. C. Technical papers are desired for the sessions of this convention. The following fields are suggested:

Reconnaissance Systems	Space Navigation	Electronic Propulsion
Ranging and Tracking	Data Handling	Communication Systems
Guidance and Control	Inertial Systems	Satellite Electronics
Military Implications of the Space Age	Simulation	Reliability (Systems)
	Instrumentation	Fire Control and Fuzing

As in the past, it is planned to hold a series of classified sessions, under military sponsorship. For these, papers limited to confidential will be accepted, with each author responsible for obtaining proper clearances.

It is planned to publish Convention Proceedings early enough to permit distribution at the time of the convention. Hence, compliance with instructions and due dates for submission of abstracts and papers will be essential. Potential authors should submit three copies of the following data by February 15, 1959: title and classification of proposed paper; abstract (unclassified—up to 250 words); author(s)' name, position, title, company affiliation, and biographical sketch.

The Technical Program Committee will notify authors of the programming of their papers early in March, at which time detailed information on standardized preparation will also be furnished so as to permit photo-offset reproduction in the Convention Proceedings. Completed papers will be required by April 10.

All papers and correspondence should be submitted to L. R. Everingham, Radiation, Inc., P.O. Box 6904, Orlando, Fla.

INFORMATION FOR AUTHORS

The PGMIL TRANSACTIONS is intended to bridge the gap between the various disciplines contributing to military electronics. Since this includes most of the branches of electronics, the military, and many fields which are associated with but not actually within the realm of electronics, it is essential that the papers published be of broad interest. The emphasis should be on readable, thought-provoking material that stimulates an attitude of open mindedness and curiosity.

Papers are solicited in the following general subject categories:

Military sciences—Military science fiction, famous battles involving electronics, basic problem areas of military electronics.

Technical survey—Tutorial technical papers on radar, communications, navigation, systems and operations research, etc.

Integrating papers—Integration of concepts common to several fields, as for example, wave phenomena, information theory, etc.

Physical sciences—Fundamentals of modern physics that may influence the future of military electronics.

Mathematical concepts—Applications and implications of modern mathematical methods.

Associated subjects—Survey of fields that are neither military nor electronic but which are important to the advancement of military electronics.

Manufacturing—Industrial and military problems of reliability, quality control, etc.

It is requested that each paper be submitted in duplicate. Standard IRE practice should be followed in preparation of the manuscript and illustrations. Papers should be sent to James Q. Brantley, Jr., or Donald R. Rhodes, PGMIL Editors, P.O. Box 6904, Orlando, Fla.

INVITATION TO MEMBERSHIP IN PGMIL

Members of the IRE may join the Professional Group on Military Electronics as active, voting members by payment of the annual fee of $2.00. Nonmembers of the IRE who qualify may become nonvoting affiliates under the new IRE affiliate Plan by payment of an annual fee of $4.50 in addition to the assessment of the Group. All applications for membership affiliation should be addressed to the Chairman of the PGMIL Membership Committee, William M. Richardson, The Ramo-Wooldridge Corporation, 1300 Connecticut Ave., Washington 6, D.C., or to IRE Headquarters.

INSTITUTIONAL LISTINGS

The IRE Professional Group on Military Electronics is grateful for the assistance given by the firms listed below, and invites application for Institutional Listings from other firms interested in the field of Military Electronics.

AIRCRAFT RADIO CORPORATION, Boonton, N.J.
Airborne Electronic Equipment and Associated Test Equipment

AVCO MANUFACTURING CORP., CROSLEY DIV., 1329 Arlington St., Cincinnati 25, Ohio
Specialists in Research, Development, Manufacture of Armament and Electronic Systems and Components

HOFFMAN SEMICONDUCTOR DIV., HOFFMAN ELECTRONICS CORP., 930 Pitner Ave., Evanston, Ill.
Silicon Alloyed-Diffused Junction Diodes & Rectifiers, Zener Reference Elements, Computer Diodes, Solar Cells

PHILCO CORP., Government and Industrial Div., 4700 Wissahickon Ave., Philadelphia 44, Pa.
Microwave, Radar, Computer, Guided Missile and Other Military Electronics Production, Research and Engineering

THE RAMO-WOOLDRIDGE CORPORATION, 5730 Arbor Vitae St., Los Angeles 45, Calif.

REPUBLIC AVIATION CORPORATION, Farmingdale, N.Y.
Aircraft and Missile Design and Manufacture

TEXAS INSTRUMENTS, INC., 6000 Lemmon Ave., Dallas 9, Texas
Radar, Sonar, M.A.D., Infrared, and Other Electronic and Electromechanical Apparatus and Systems

VARIAN ASSOCIATES, 611 Hansen Way, Palo Alto, Calif.
Klystrons, BWOs, TWTs, Stalos, UHF Waterloads, Microwave Components, Research and Development Services

The charge for an Institutional Listing is $75.00 per issue or $225.00 for four consecutive issues. Applications for Institutional Listings and checks (made out to the Institute of Radio Engineers) should be sent to Mr. L. G. Cumming, Technical Secretary, Institute of Radio Engineers, 1 East 79th Street, New York 21, N. Y.